

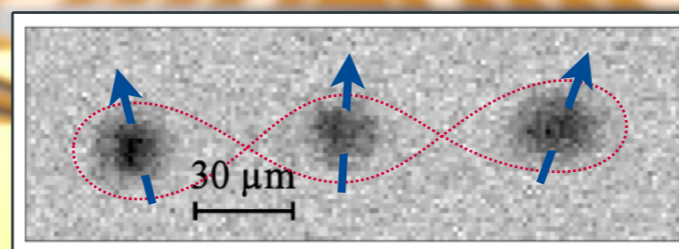
Multiparameter quantum metrology with EPR entangled BECs

Philipp Treutlein

atom.physik.unibas.ch

Colciaghi et al, PRX 13, 021031 (2023)

Atom chip



Entangled atomic
Bose condensates

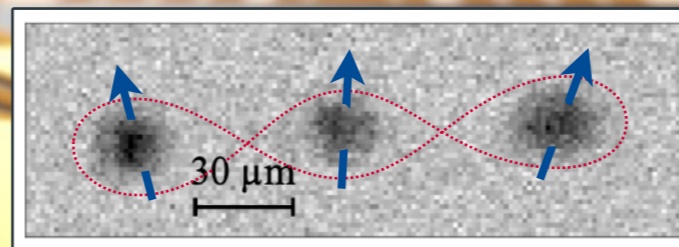
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Spin squeezing

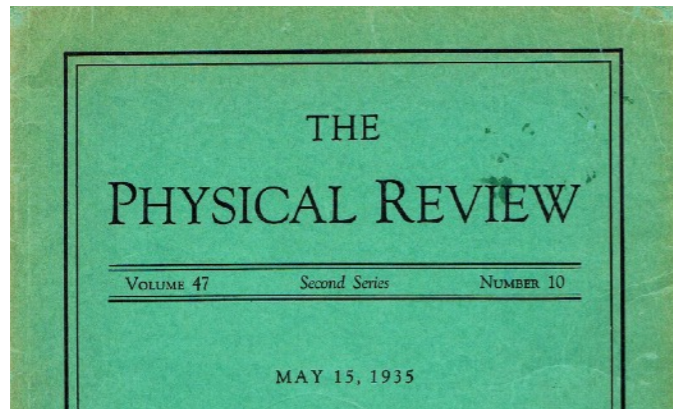


Entangled atomic
Bose condensates

Quantum foundations

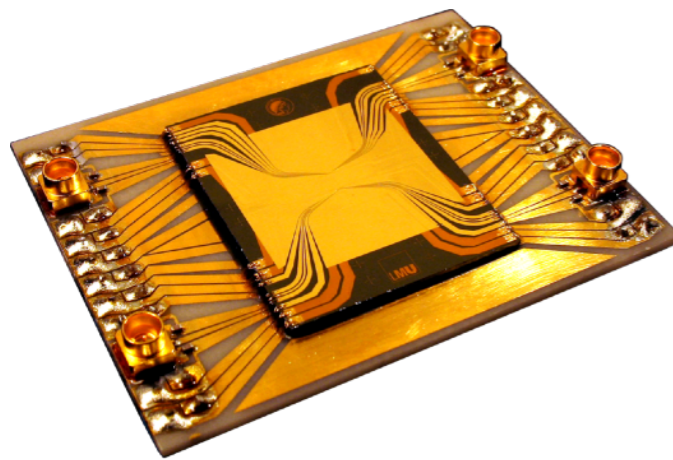
Quantum metrology

Outline



The Einstein-Podolsky-Rosen paradox

Einstein, Podolsky, Rosen, Phys Rev 47, 777 (1935)



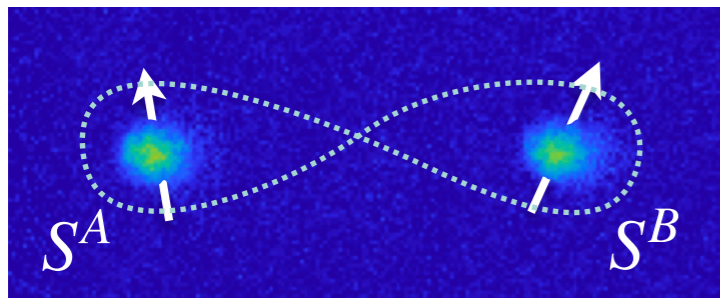
Two-component Rb BEC on atom chip Spin-squeezing, quantum metrology

Riedel et al, Nature 464, 1170 (2010)

Ockeloen et al, PRL 111, 143001 (2013)

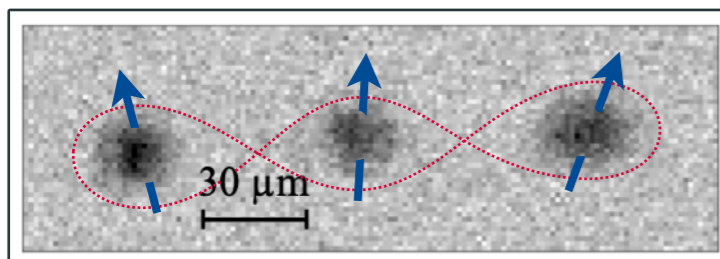
Schmied et al, Science 352, 441 (2016)

Fadel et al, Science 360, 409 (2018)



EPR paradox between two spatially separated and addressable BECs

Colciaghi et al, PRX 13, 021031 (2023)



Multiparameter estimation with an array of entangled atomic sensors

collaboration: Y. Baamara, A. Sinatra

The Einstein-Podolsky-Rosen paradox



Heisenberg uncertainty relation for system B:

$$\text{Var}(\hat{S}_y^B) \text{Var}(\hat{S}_z^B) \geq \frac{1}{4} \left| \langle \hat{S}_x^B \rangle \right|^2$$

Entanglement: measurement outcomes in A allow to predict (“infer”) measurement outcomes in B

$$\hat{S}_{y,\text{inf}}^B = g_y \hat{S}_y^A + c_y \quad \text{linear estimate}$$

$$\text{Var}_{\text{inf}}(\hat{S}_y^B) = \text{Var}(\hat{S}_y^B - \hat{S}_{y,\text{inf}}^B) \quad \text{“inferred variance”}$$

EPR paradox

$$\text{Var}_{\text{inf}}(\hat{S}_y^B) \text{Var}_{\text{inf}}(\hat{S}_z^B) < \frac{1}{4} \left| \langle \hat{S}_x^B \rangle \right|^2$$

Inferred uncertainty product below the Heisenberg relation

Einstein, Podolsky, and Rosen, Phys. Rev. 47, 777 (1935)
 Schrödinger, Math. Proc. Cambridge Philos. Soc. 31, 555 (1935)
 M. D. Reid et al, Rev. Mod. Phys. 81, 1727 (2009)

EPR steering and entanglement



EPR criterion

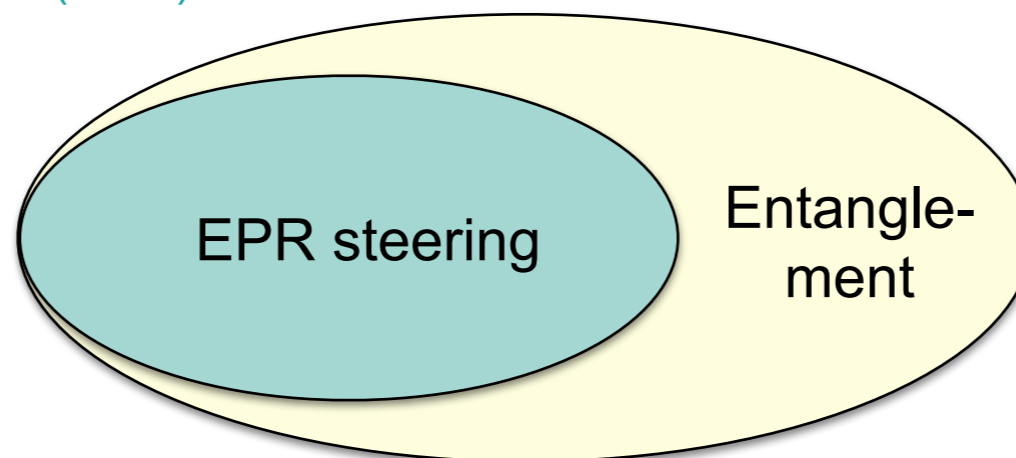
$$E_{EPR}^{A \rightarrow B} = \frac{4 \text{Var}(\hat{S}_y^B - g_y \hat{S}_y^A) \text{Var}(\hat{S}_z^B - g_z \hat{S}_z^A)}{|\langle \hat{S}_x^B \rangle|^2} < 1$$

M. D. Reid, PRA 40, 913 (1989)

Entanglement criterion

$$E_{Ent} = \frac{4 \text{Var}(\hat{S}_y^B - g_y \hat{S}_y^A) \text{Var}(\hat{S}_z^B - g_z \hat{S}_z^A)}{\left(|\langle \hat{S}_x^B \rangle| + |g_y g_z| |\langle \hat{S}_x^A \rangle| \right)^2} < 1$$

Giovannetti et al, PRA 67, 022320 (2003)

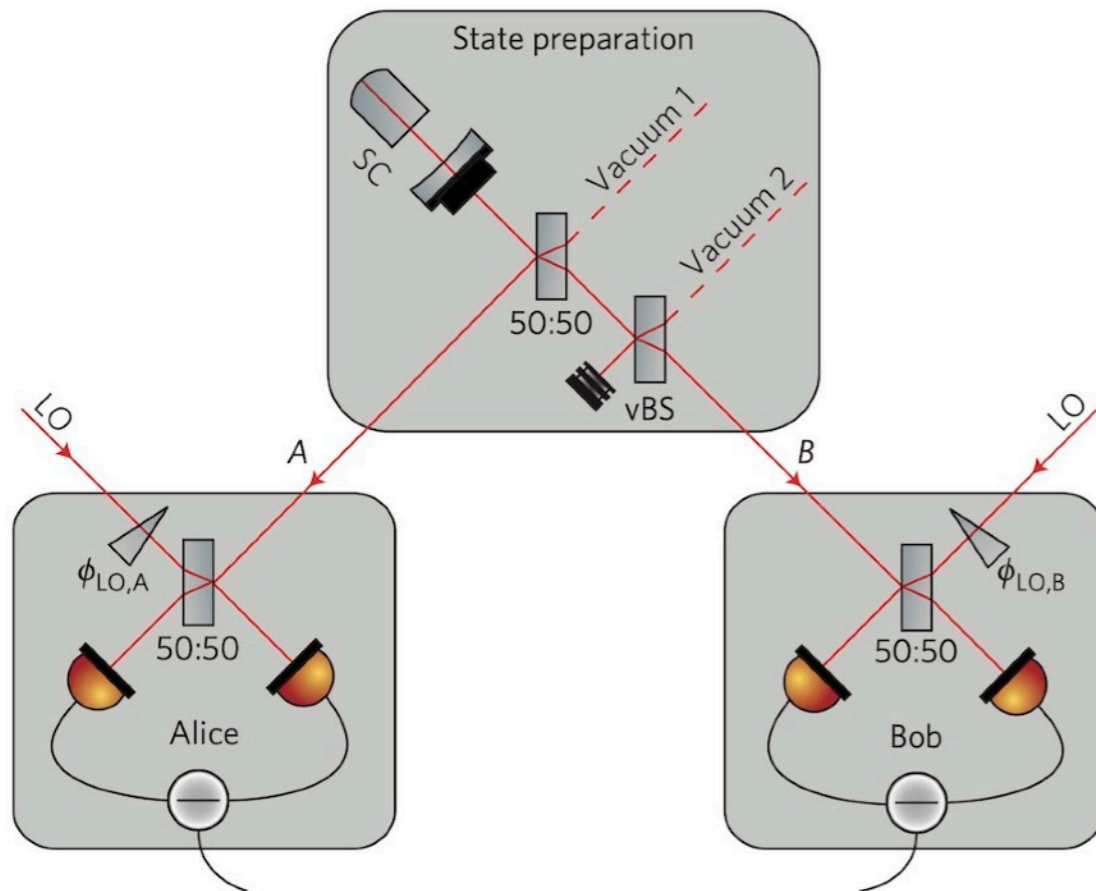


Wiseman et al, PRL 98, 140402 (2007)

Experiments in optics: Ou et al, PRL 68, 3663 (1992), Silberhorn et al, PRL 86, 4267 (2001), Schori et al, PRA 66, 033802 (2002), Bowen et al, PRL 90, 043601 (2003), ...

EPR experiments

Demonstrations of the EPR paradox



Optics

- Ou et al, PRL 68, 3663 (1992)
- Bowen et al, PRL 90, 043601 (2003)
- Händchen et al, Nat Photon 6, 598 (2012)
- Reid et al, Rev Mod Phys 81, 1727 (2009)

Bell tests

Bell, Physics 1, 195 (1964)

- 2 photons, 4 photons
- 2 ions, up to 14 ions
- 2 neutral atoms
- 2 superconducting qubits
- 2 solid-state spin qubits

Freedman, Clauser, PRL 28, 938 (1972)

Aspect et al, PRL 49, 1804 (1982)

Weih's et al, PRL 81, 5039 (1998)

...

Hensen et al, Nature 526, 682 (2015)

Giustina et al, PRL 115, 250401 (2015)

Shalm et al, PRL 115, 250402 (2015)

Rosenfeld et al, PRL 119, 195 (2017)

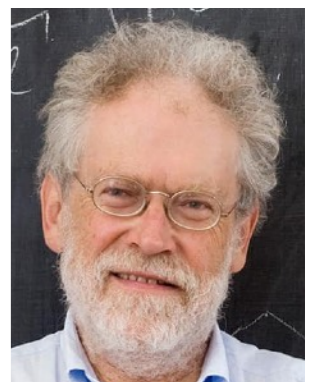
**The EPR paradox and nonlocality
are firmly established in
few-particle systems!**



A. Aspect



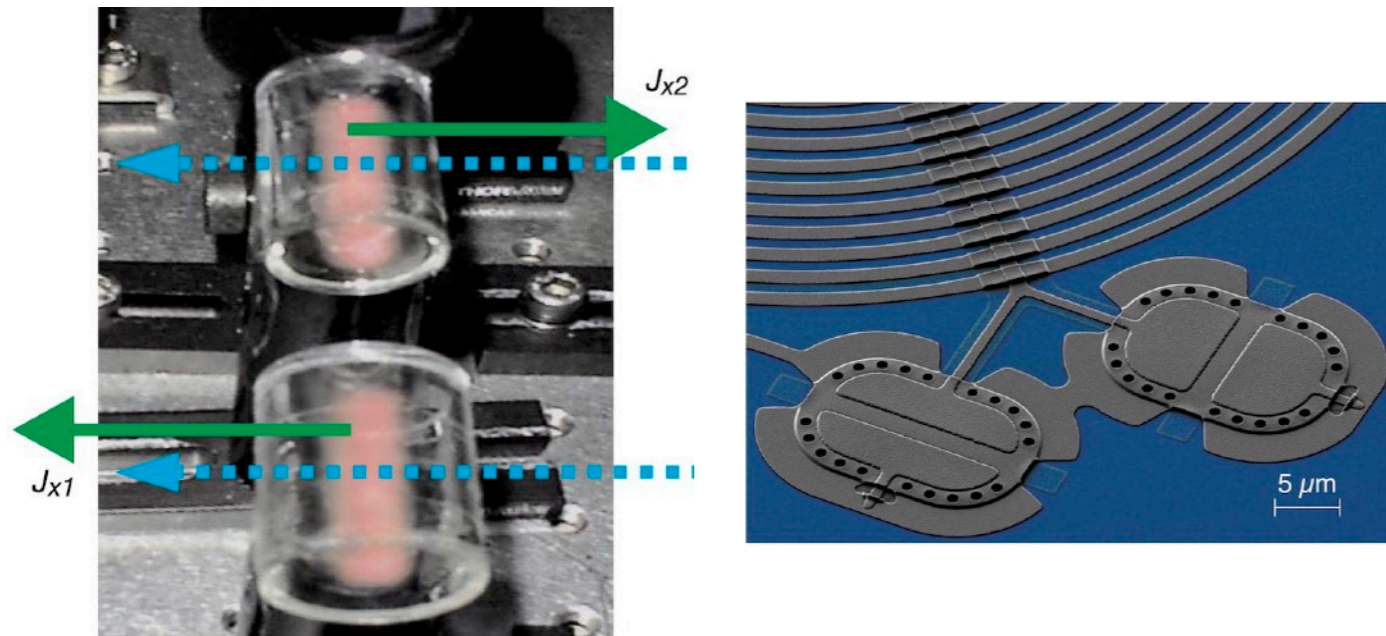
J. F. Clauser



A. Zeilinger

Entanglement and EPR in massive many-body systems

Entanglement between spatially separated systems



Atomic ensembles

Julsgaard et al, Nature 413, 400 (2001)
Chou et al, Nature 438, 828 (2005)
Lange et al, Science 360, 416 (2018)

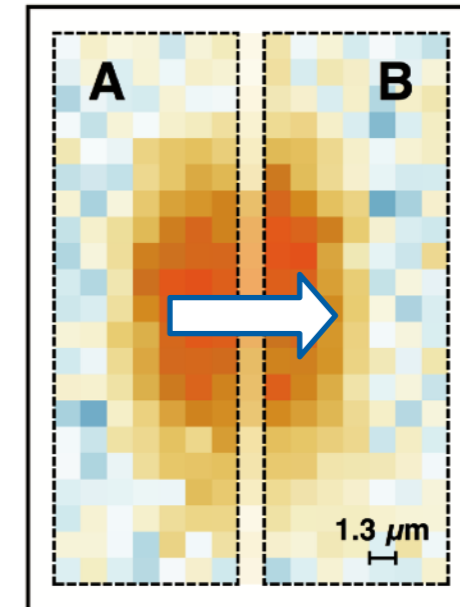
Mechanical oscillators

Kotler et al, Science 372, 622 (2021)
de Lépinay, Science 372, 625 (2021)

Hybrid systems

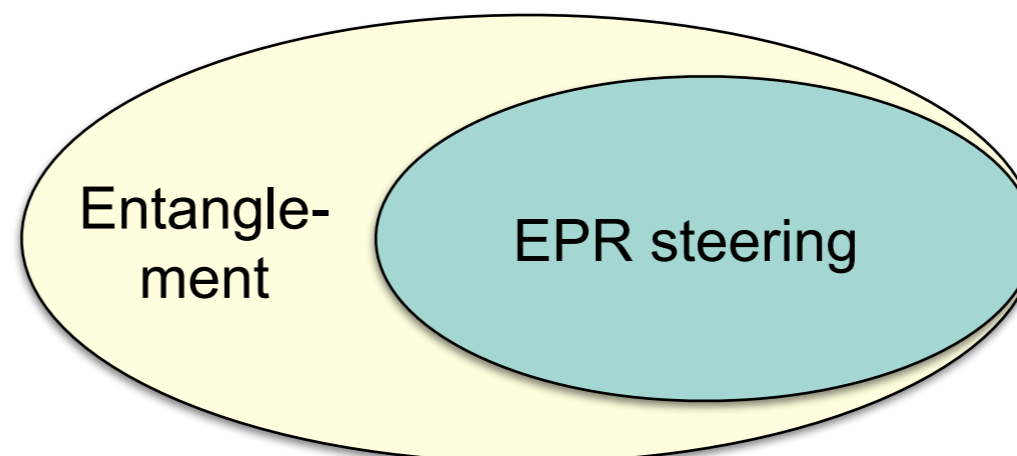
Lettner et al, PRL 106, 210503 (2011)
Thomas et al, Nat Phys 17, 228 (2021)

EPR steering in a single system

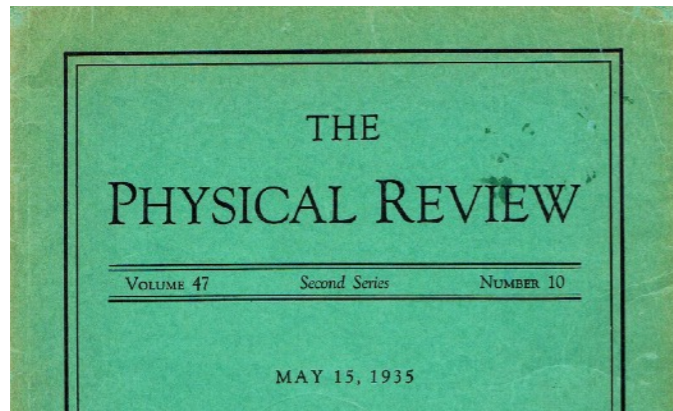


Atomic ensembles

Peise et al, Nat. Comm. 6, 8984 (2015)
Fadel et al, Science 360, 409 (2018)
Kunkel et al, Science 360, 413 (2018)

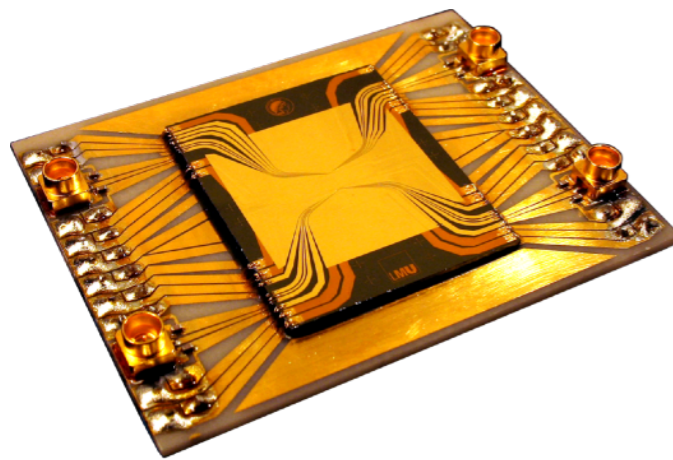


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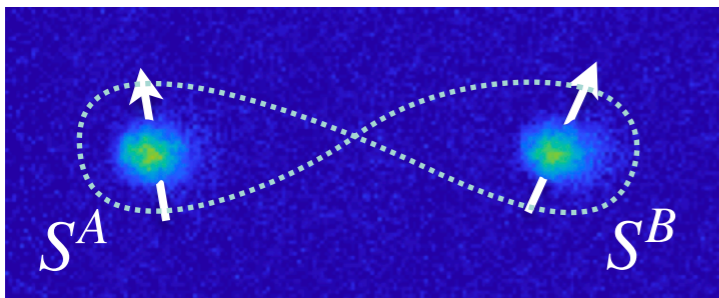
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Riedel et al, Nature 464, 1170 (2010)

Ockeloen et al, PRL 111, 143001 (2013)

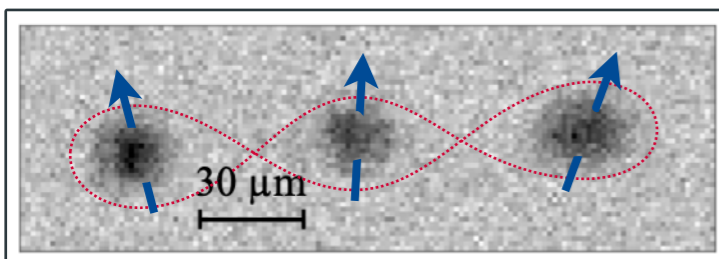
Schmied et al, Science 352, 441 (2016)

Fadel et al, Science 360, 409 (2018)



EPR paradox between two spatially separated and addressable BECs

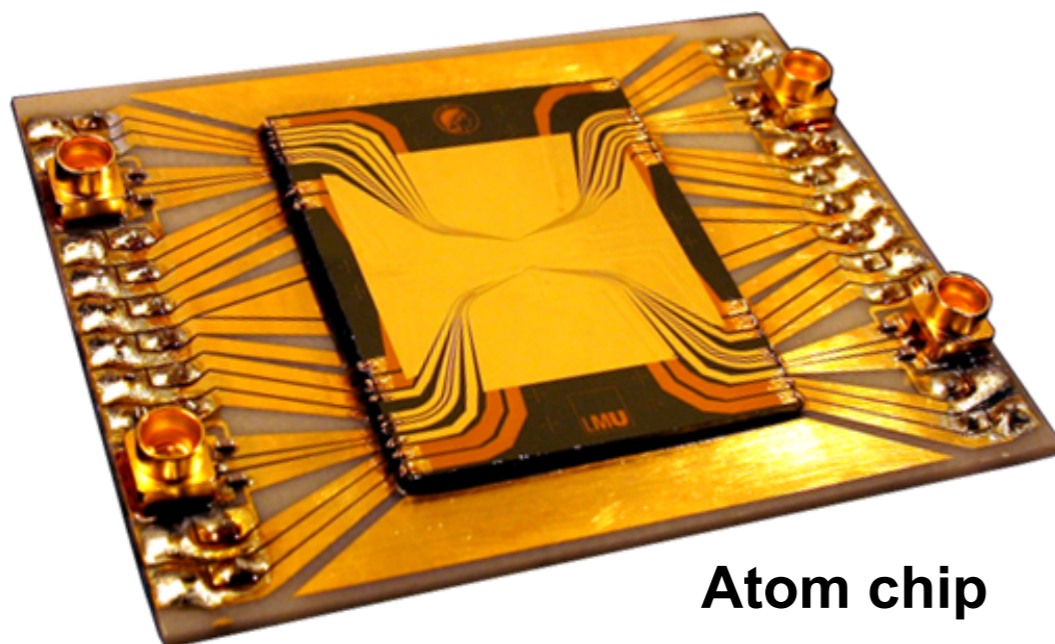
Colciaghi et al, PRX 13, 021031 (2023)



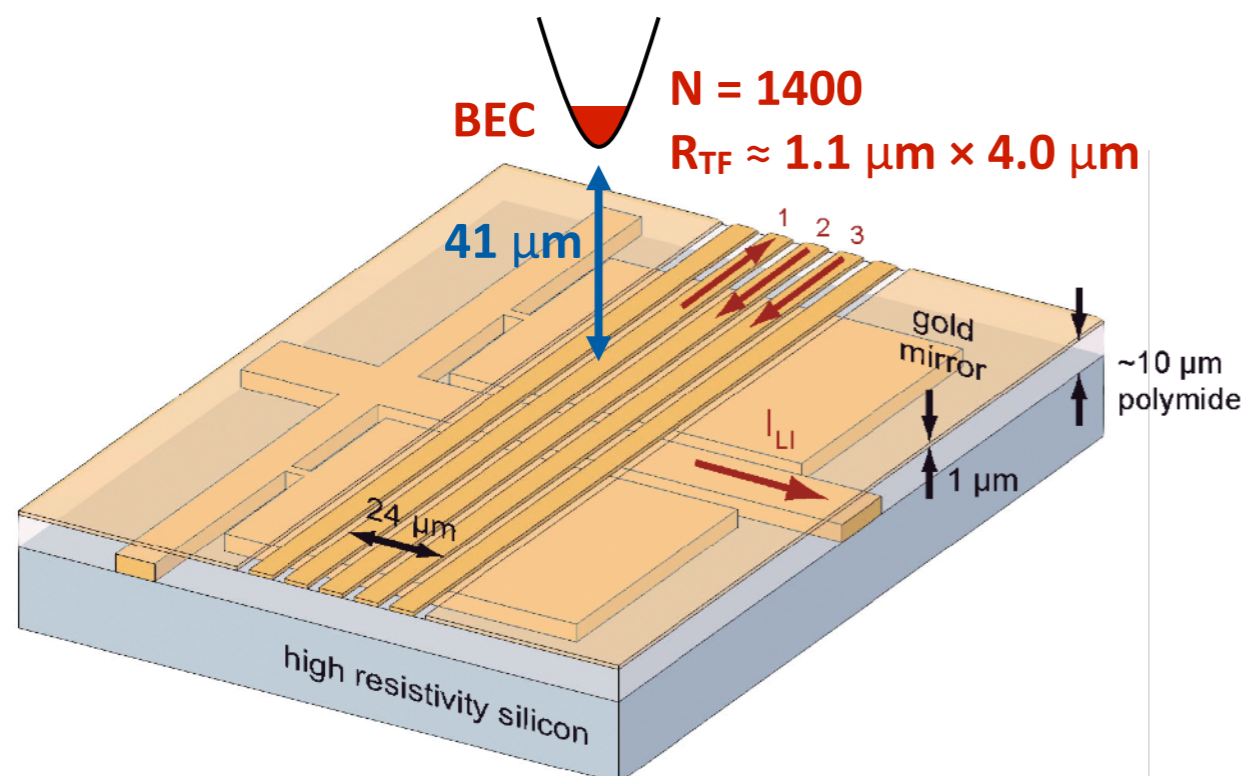
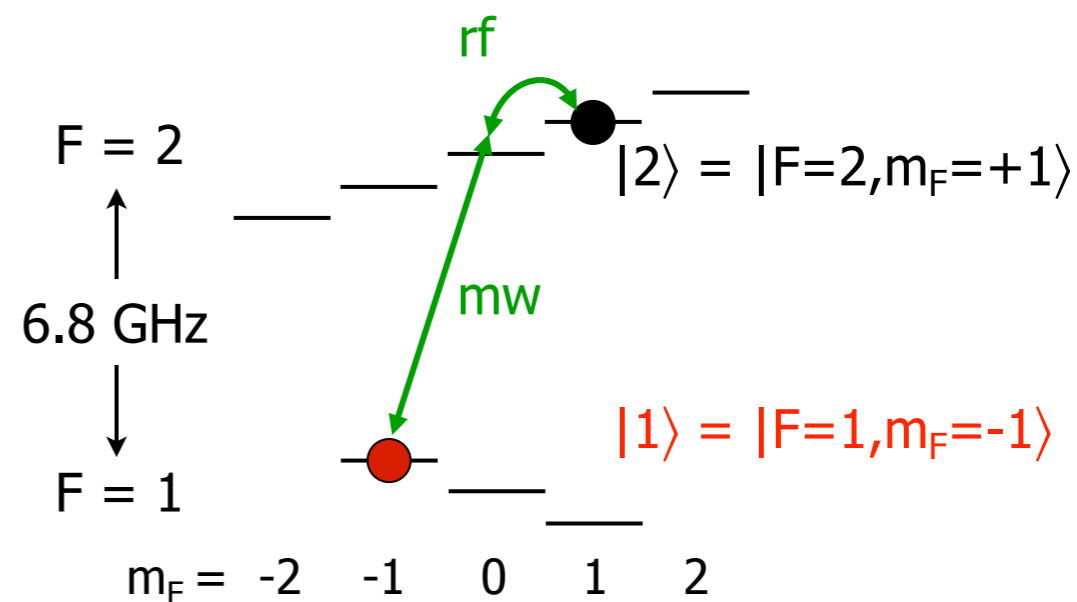
Multiparameter estimation with an array of entangled atomic sensors

collaboration: Y. Baamara, A. Sinatra

Two-component ^{87}Rb BEC on an atom chip

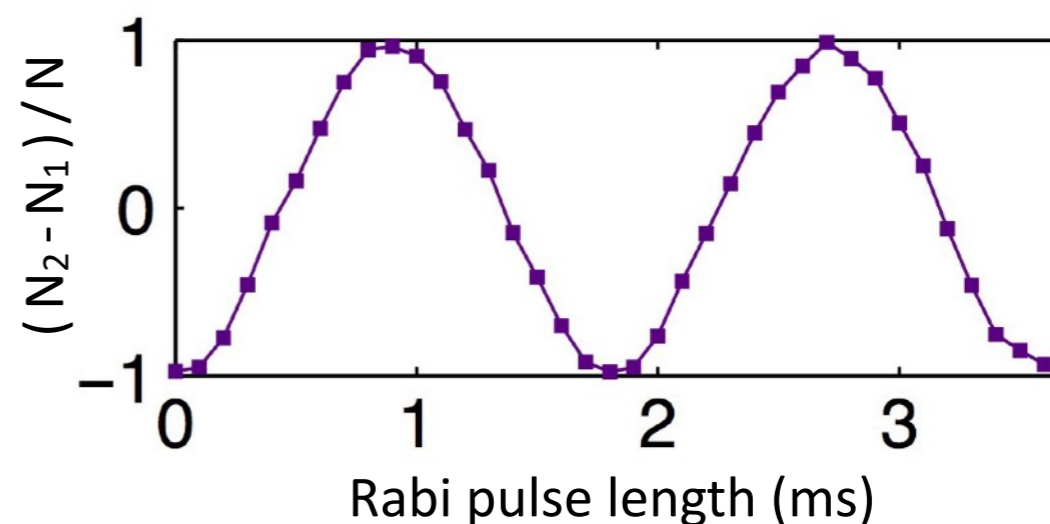


^{87}Rb ground-state hyperfine structure

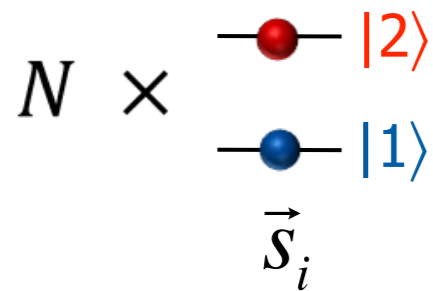
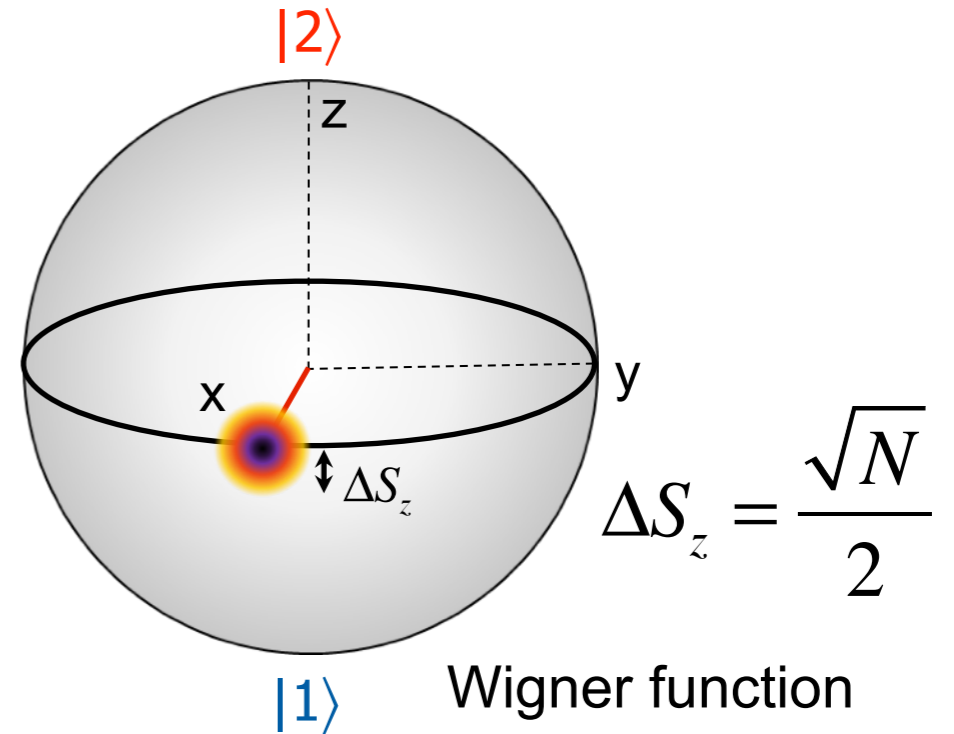
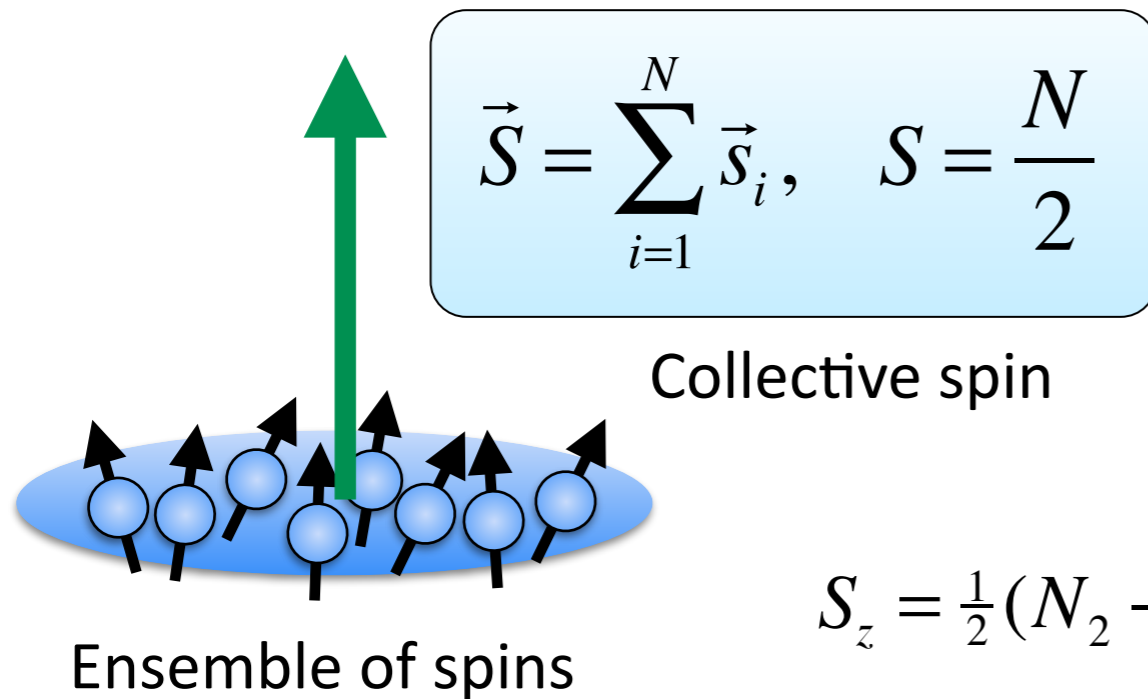


Rabi oscillations

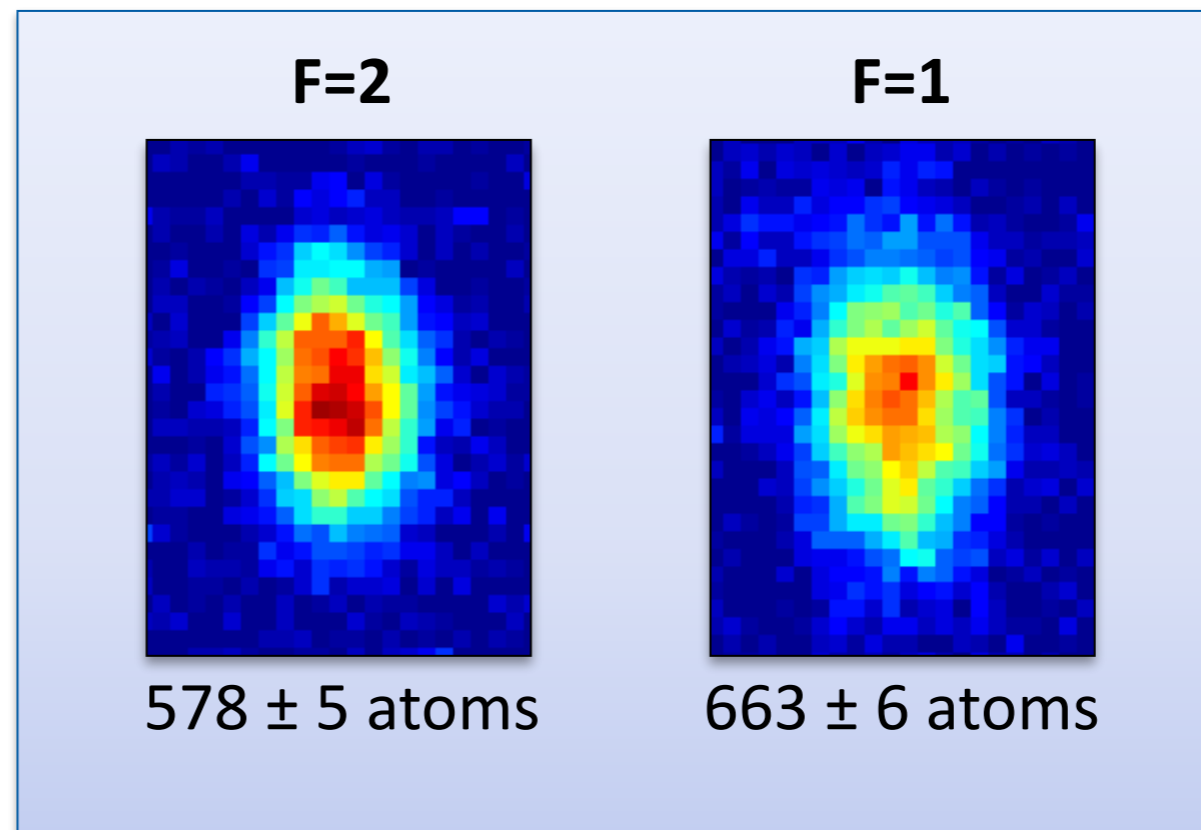
fidelity of $\pi/2$ -pulse: $(99.74 \pm 0.04)\%$



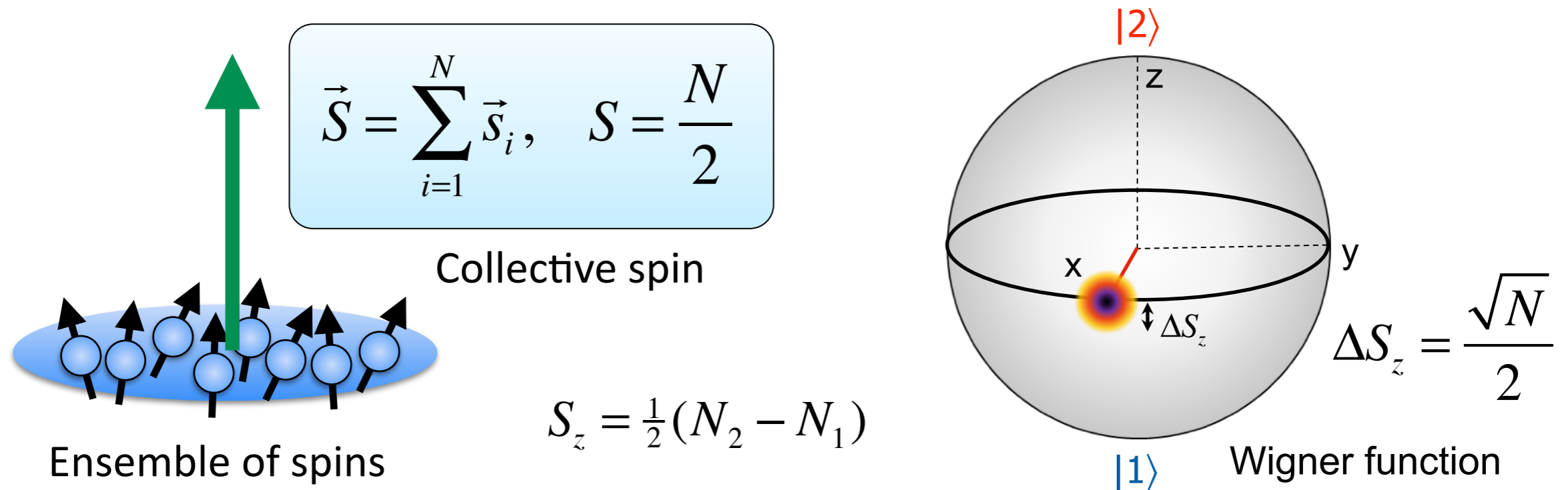
Collective spin description of BEC internal state



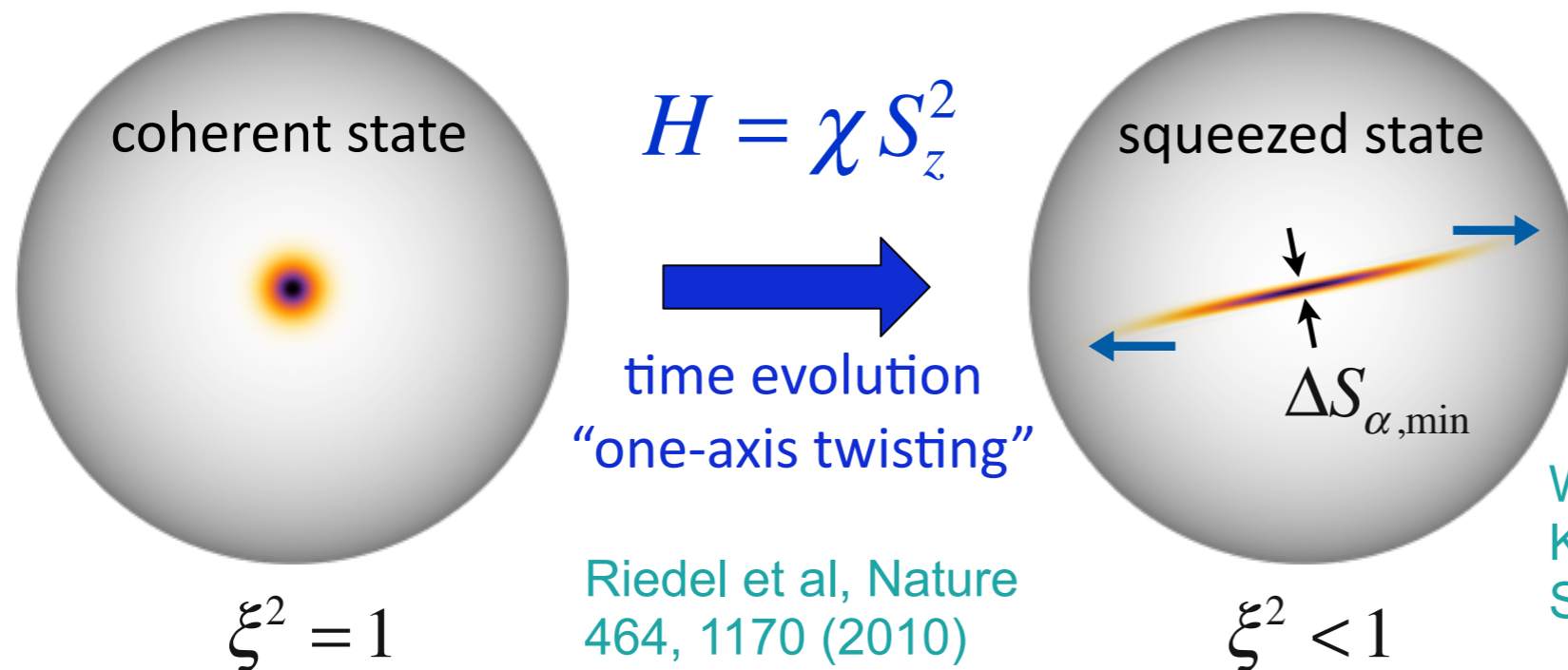
Absorption images of both states in single shot of experiment



Spin-squeezing through collisions



Atomic collisions create entanglement



Spin-squeezing parameter

$$\xi^2 \equiv \frac{N (\Delta S_{\alpha, \min})^2}{\langle S_x \rangle^2}$$

entanglement witness

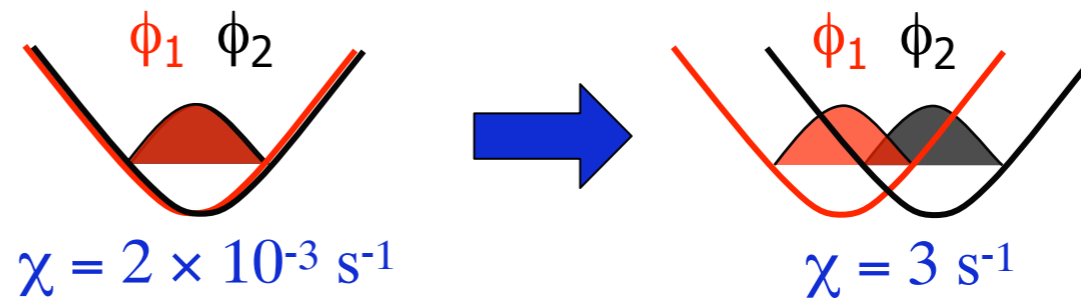
Wineland, Bollinger, Itano (1992)
 Kitagawa, Ueda (1993)
 Sørensen, Duan, Cirac, Zoller (2001)

Controlled collisions in state-dependent potentials

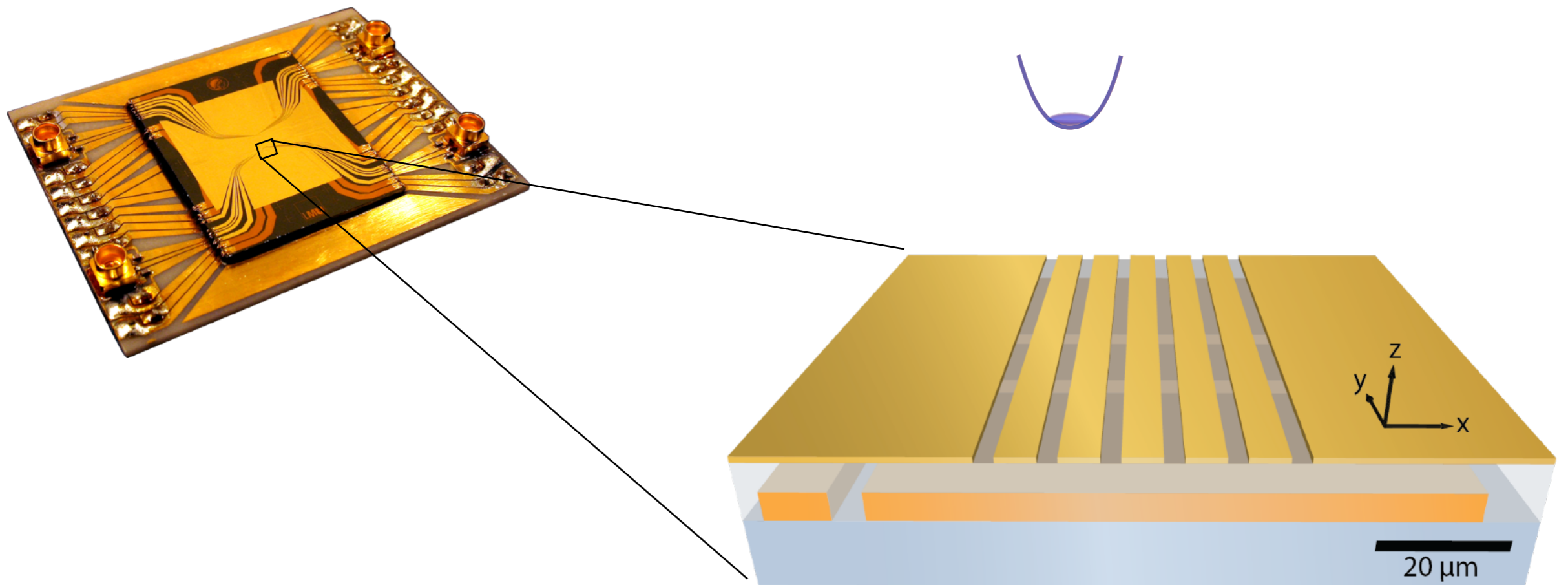
How can we implement the squeezing Hamiltonian?

$$H = \chi S_z^2$$

→ collisions between atoms in a state-dependent potential



$$\chi \sim a_{11} \int |\phi_1|^4 dr^3 + a_{22} \int |\phi_2|^4 dr^3 - 2a_{12} \int |\phi_1|^2 |\phi_2|^2 dr^3$$



Li, Treutlein, Reichel, Sinatra, Eur Phys J B 68, 365 (2009)

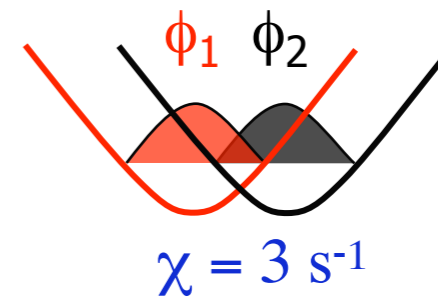
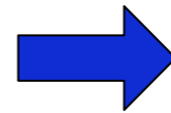
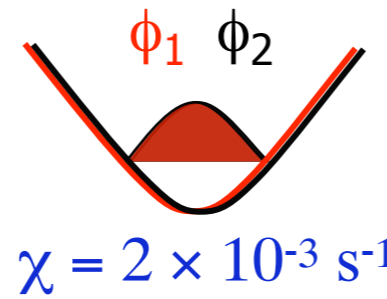
Treutlein, Hänsch, Reichel, Negretti, Cirone, Calarco, PRA 74, 022312 (2006)

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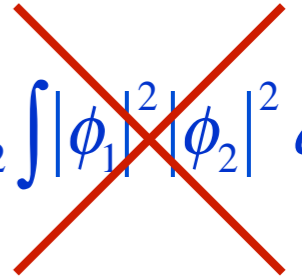
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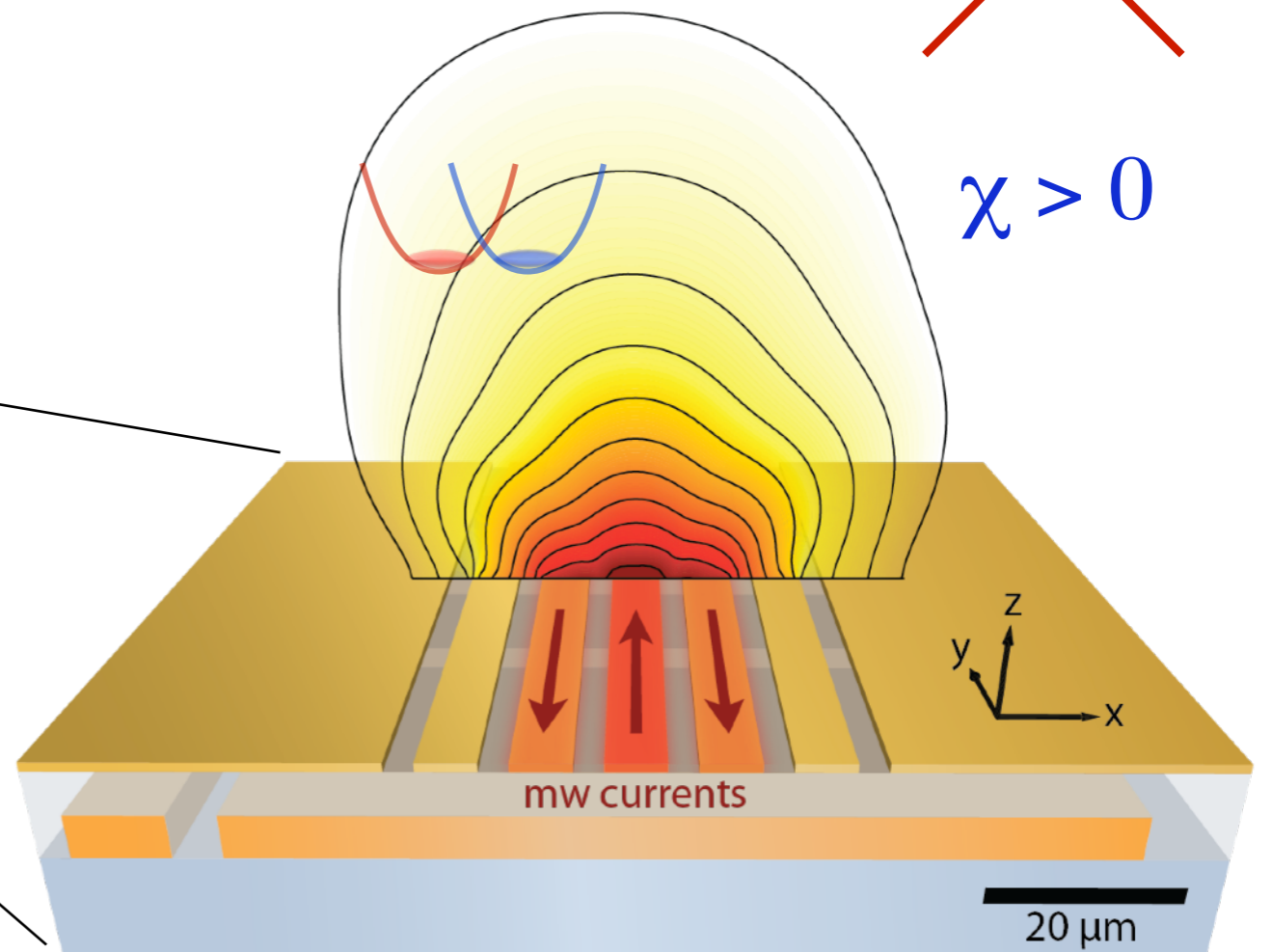
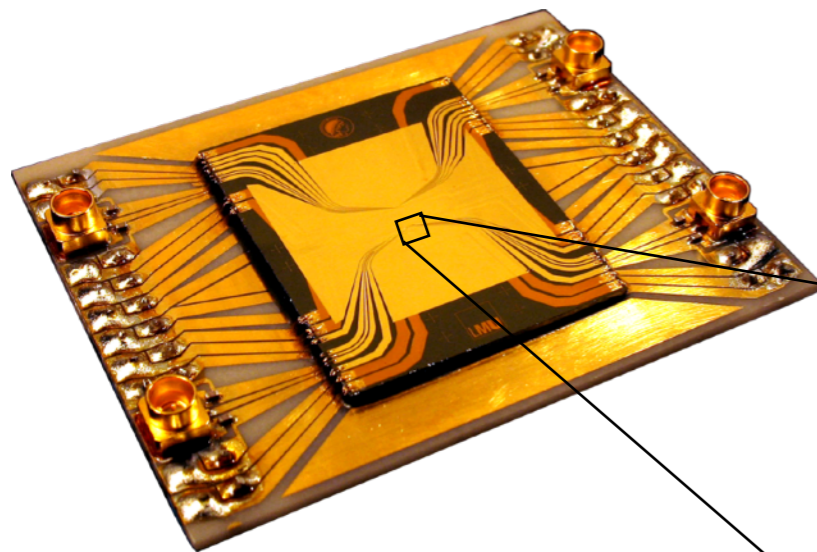
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$$\chi \sim a_{11} \int |\phi_1|^4 dr^3 + a_{22} \int |\phi_2|^4 dr^3 - 2a_{12} \int |\phi_1|^2 |\phi_2|^2 dr^3$$



$\chi > 0$



Li, Treutlein, Reichel, Sinatra, Eur Phys J B 68, 365 (2009)

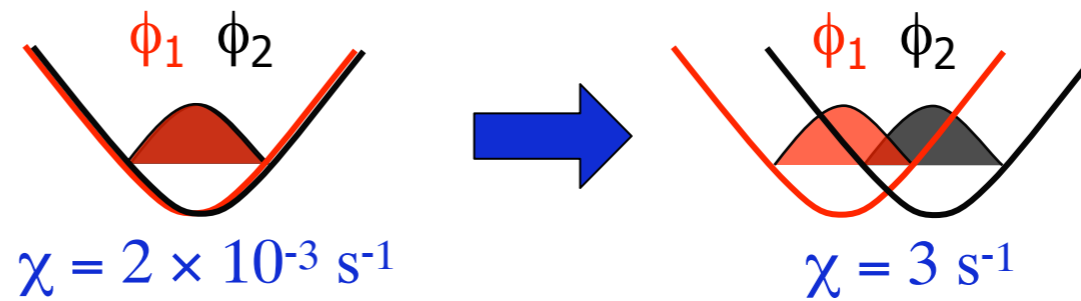
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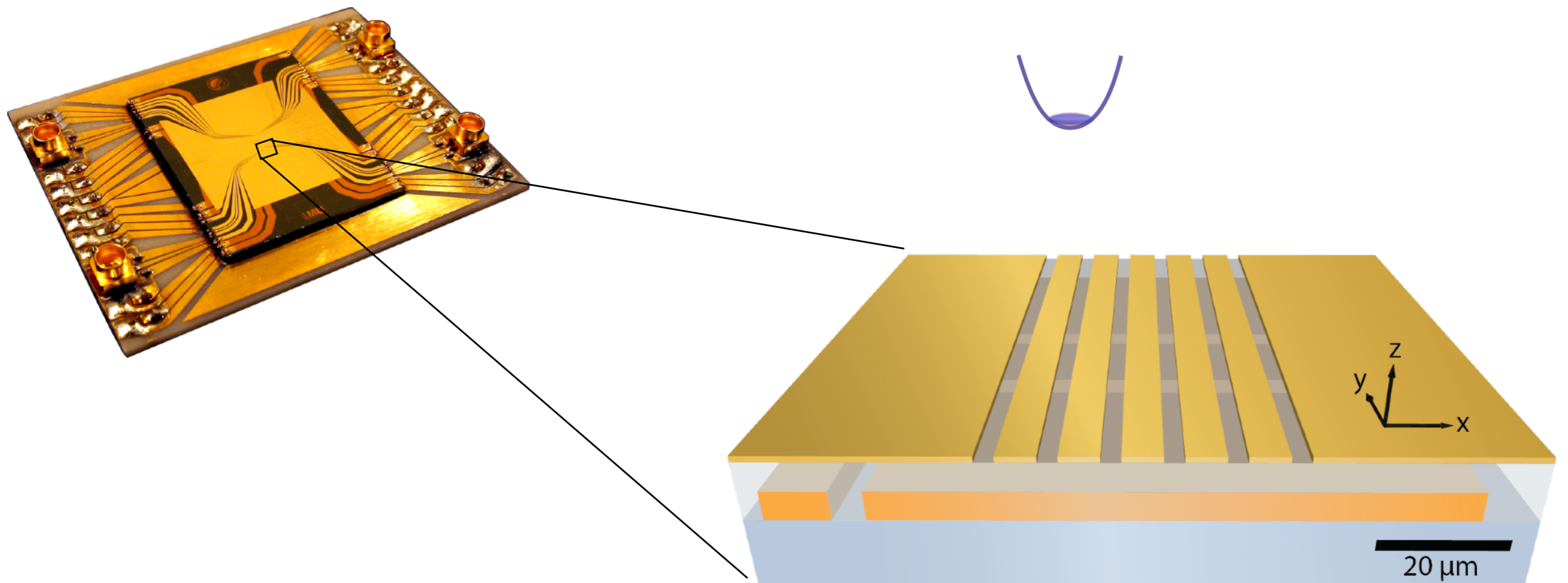
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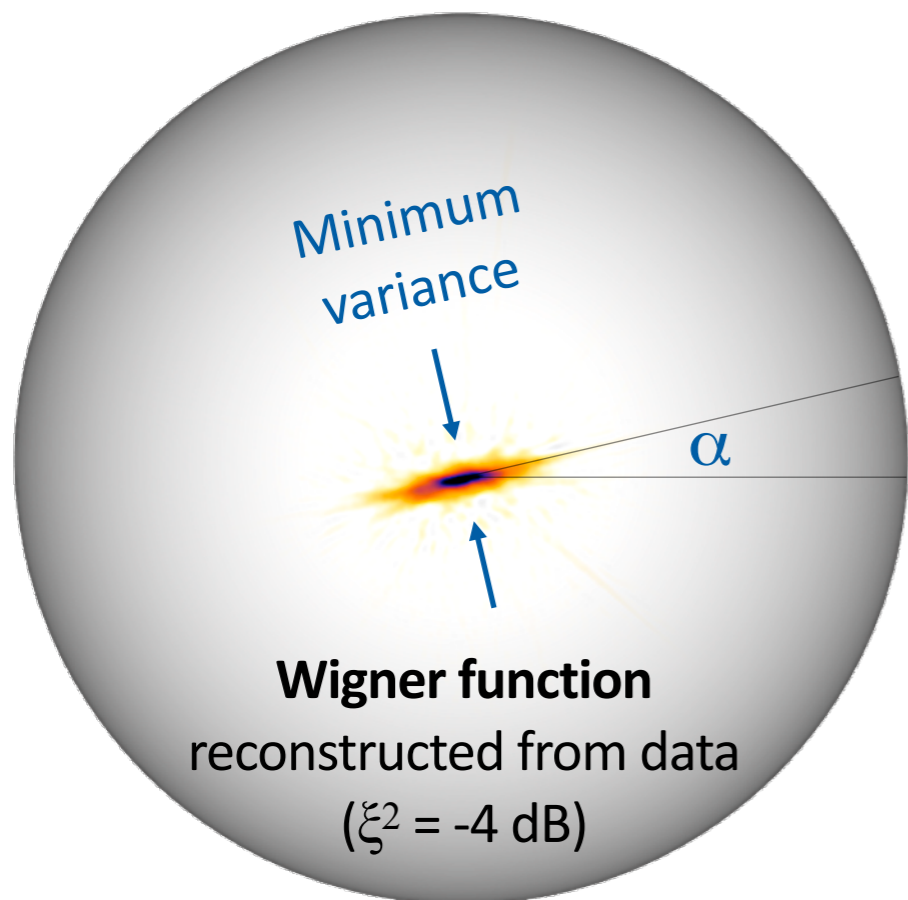
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Li, Treutlein, Reichel, Sinatra, Eur Phys J B 68, 365 (2009)

Treutlein, Hänsch, Reichel, Negretti, Cirone, Calarco, PRA 74, 022312 (2006)

Tomography of spin-squeezed state

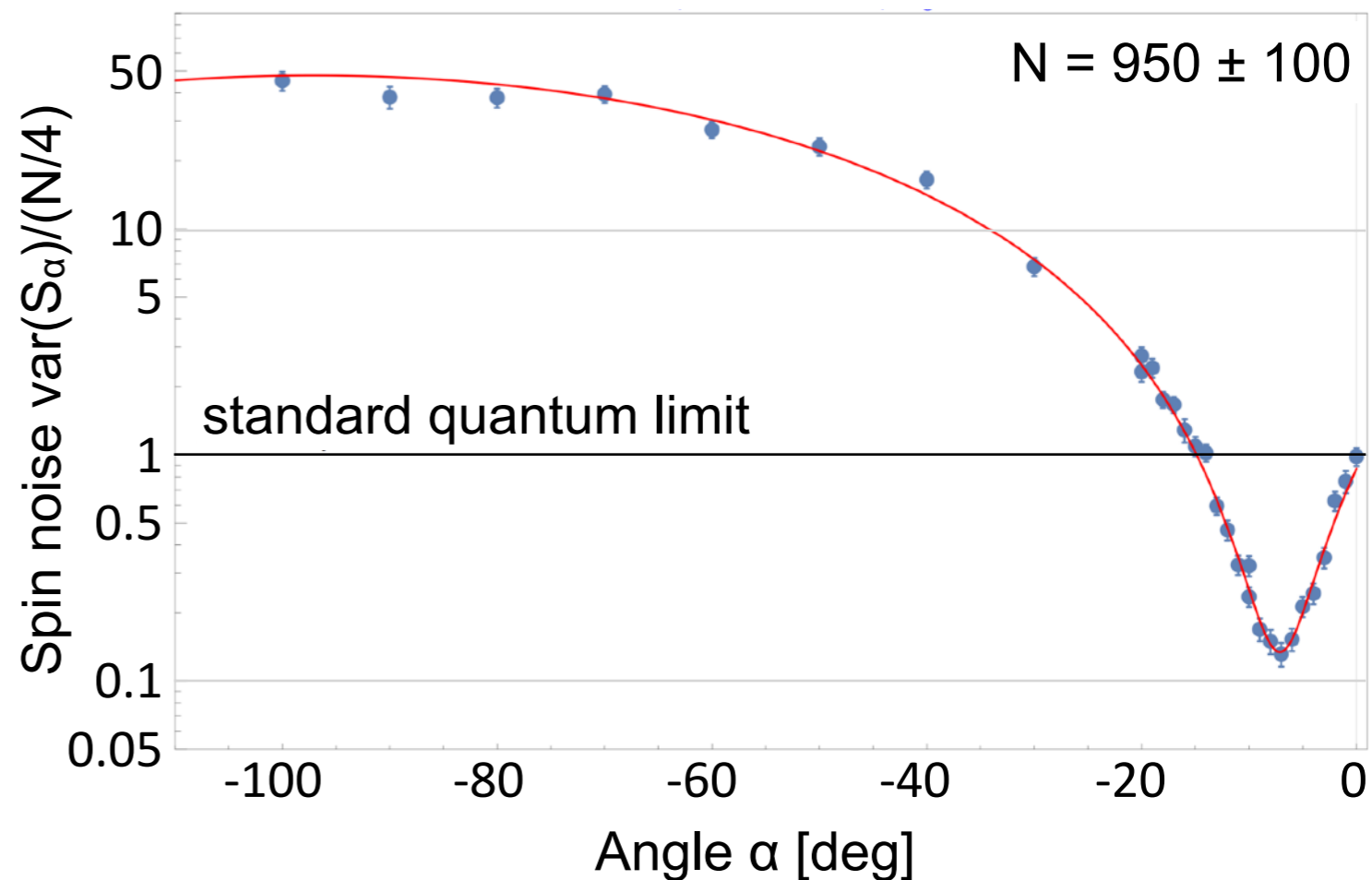


$$W(\vartheta, \varphi) = \sum_{k=0}^{2j} \sum_{q=-k}^k \rho_{kq} Y_{kq}(\vartheta, \varphi)$$

Squeezing and tomography

Riedel et al, Nature 464, 1170 (2010)

Schmied et al, New J Phys 13, 065019 (2011)



$$\xi^2 = -8.2 \pm 0.5 \text{ dB}$$

\Rightarrow entanglement

(Noise reduced by -8.7 ± 0.5 dB,
contrast $C = 94.9\%$)

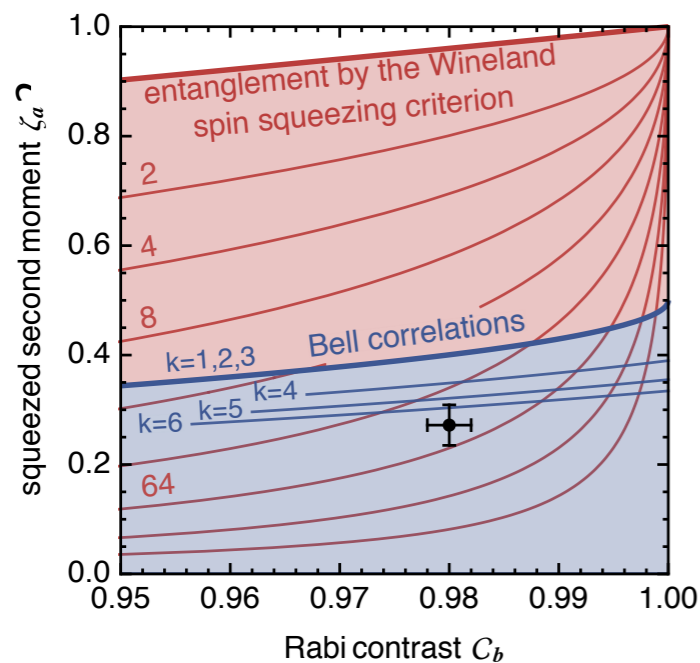
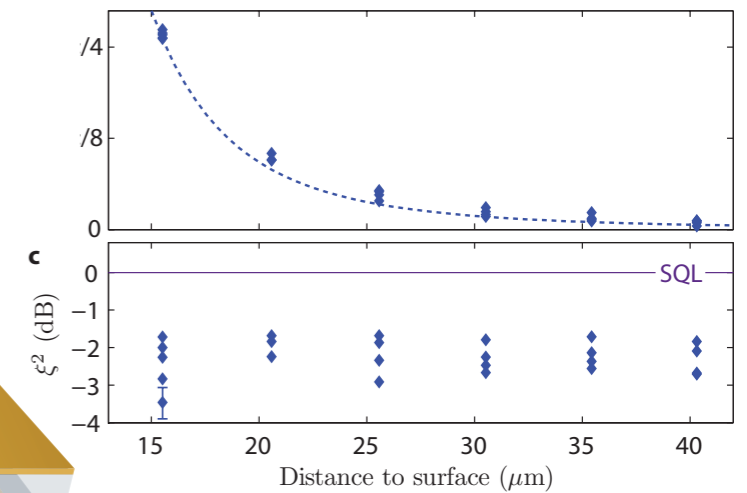
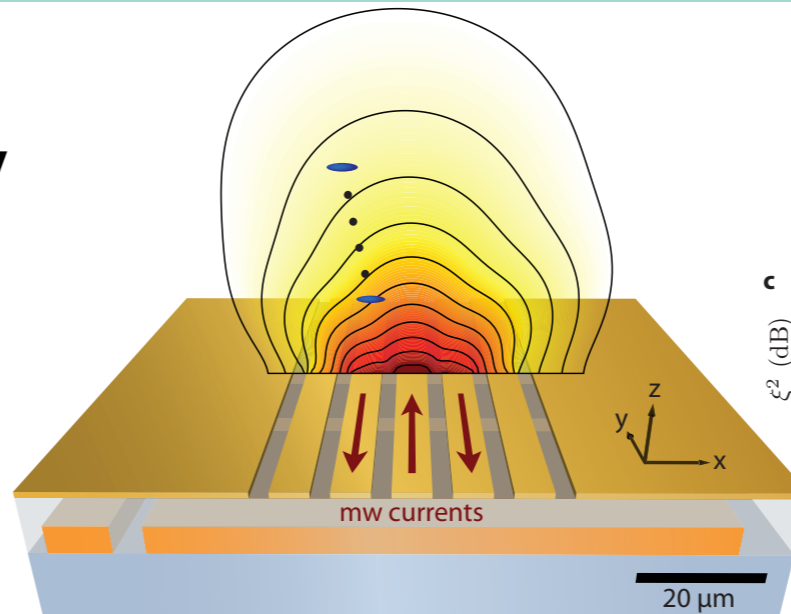
Experiments with spin-squeezed atoms

Quantum-enhanced interferometry

- mw field sensing close to chip surface
- enhancement of 7 dB beyond SQL

Riedel et al, Nature 464, 1170 (2010)

Ockeloen et al, PRL 111, 143001 (2013)



Many-particle Bell correlations

- Bell correlations in many-particle system detected by global measurements

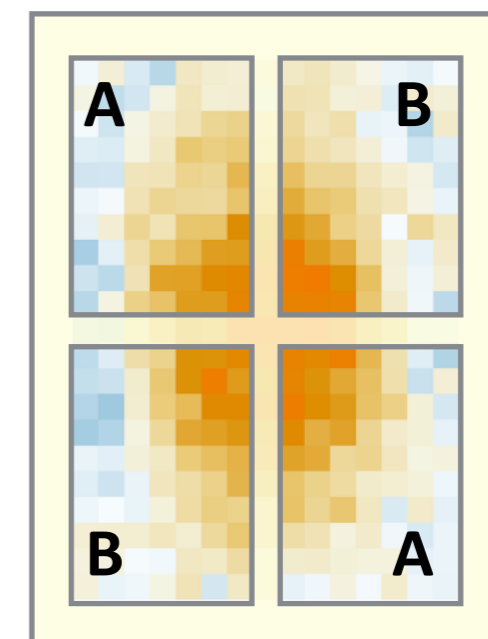
Schmied et al, Science 352, 441 (2016)

Wagner et al, PRL 119, 170403 (2017)

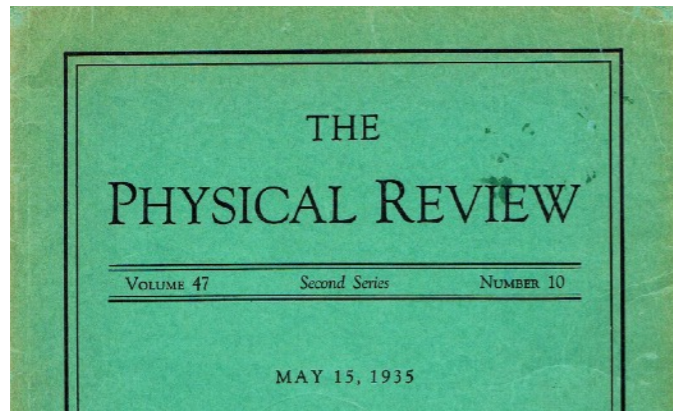
Entanglement patterns

- Entanglement and steering between parts of BEC observed by high-resolution imaging

Fadel et al, Science 360, 409 (2018)

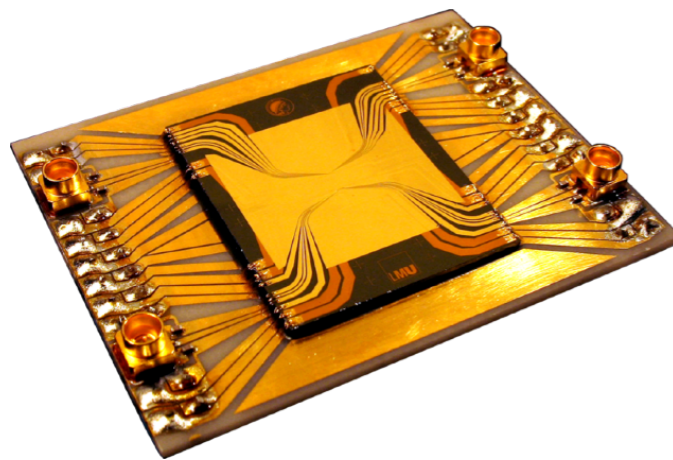


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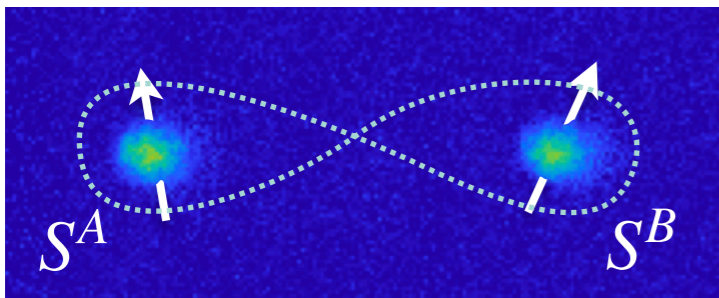
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Ockeloen et al, PRL 111, 143001 (2013)

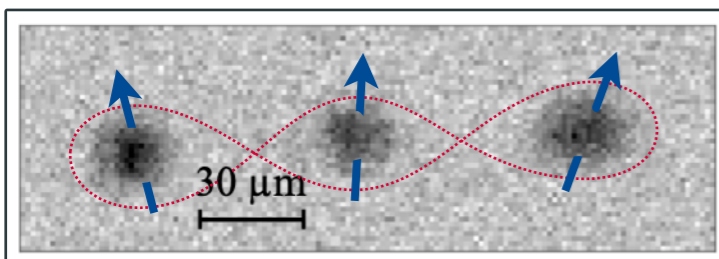
Schmied et al, Science 352, 441 (2016)

Fadel et al, Science 360, 409 (2018)



EPR paradox between two spatially separated and addressable BECs

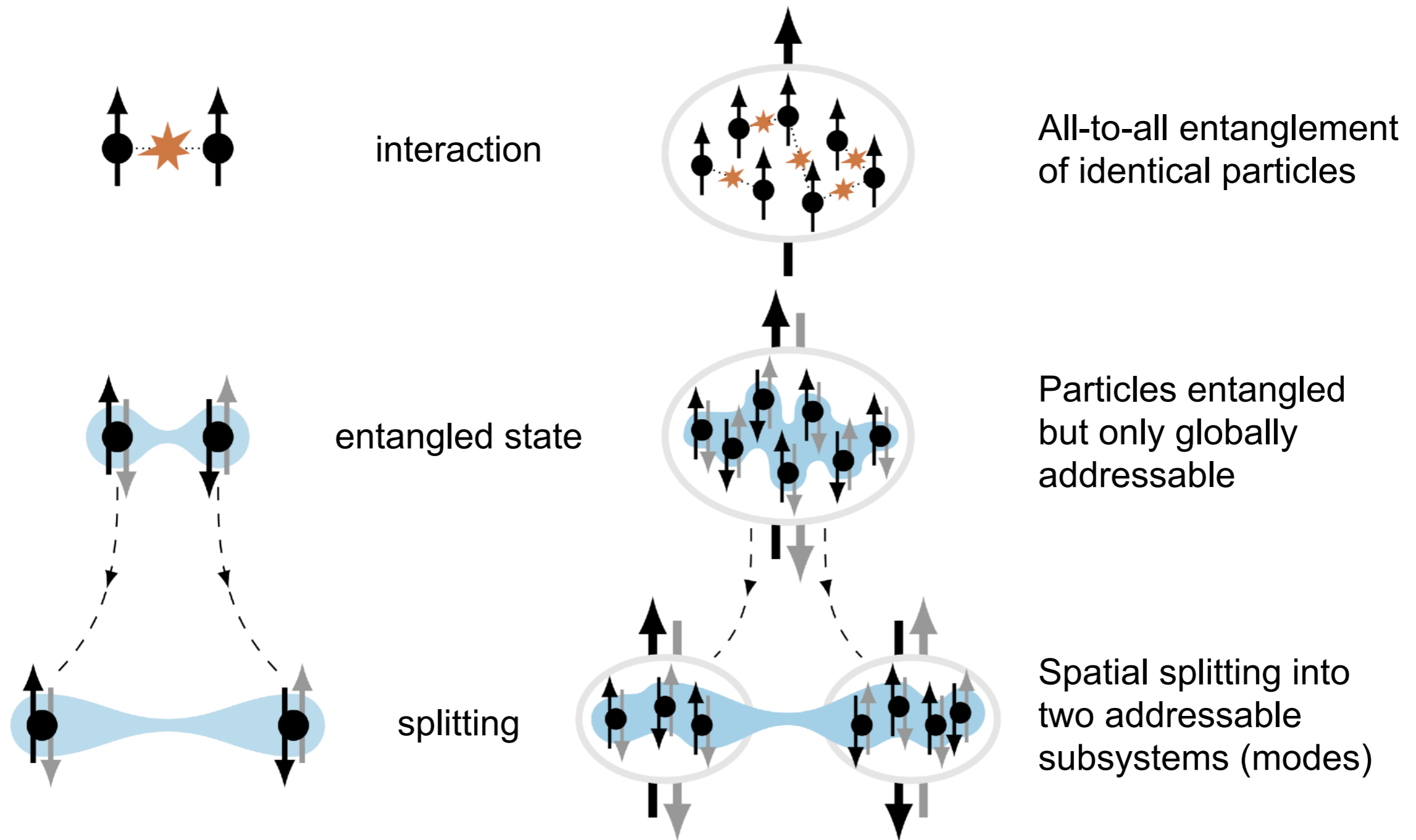
Colciaghi et al, PRX 13, 021031 (2023)



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collaboration: Y. Baamara, A. Sinatra

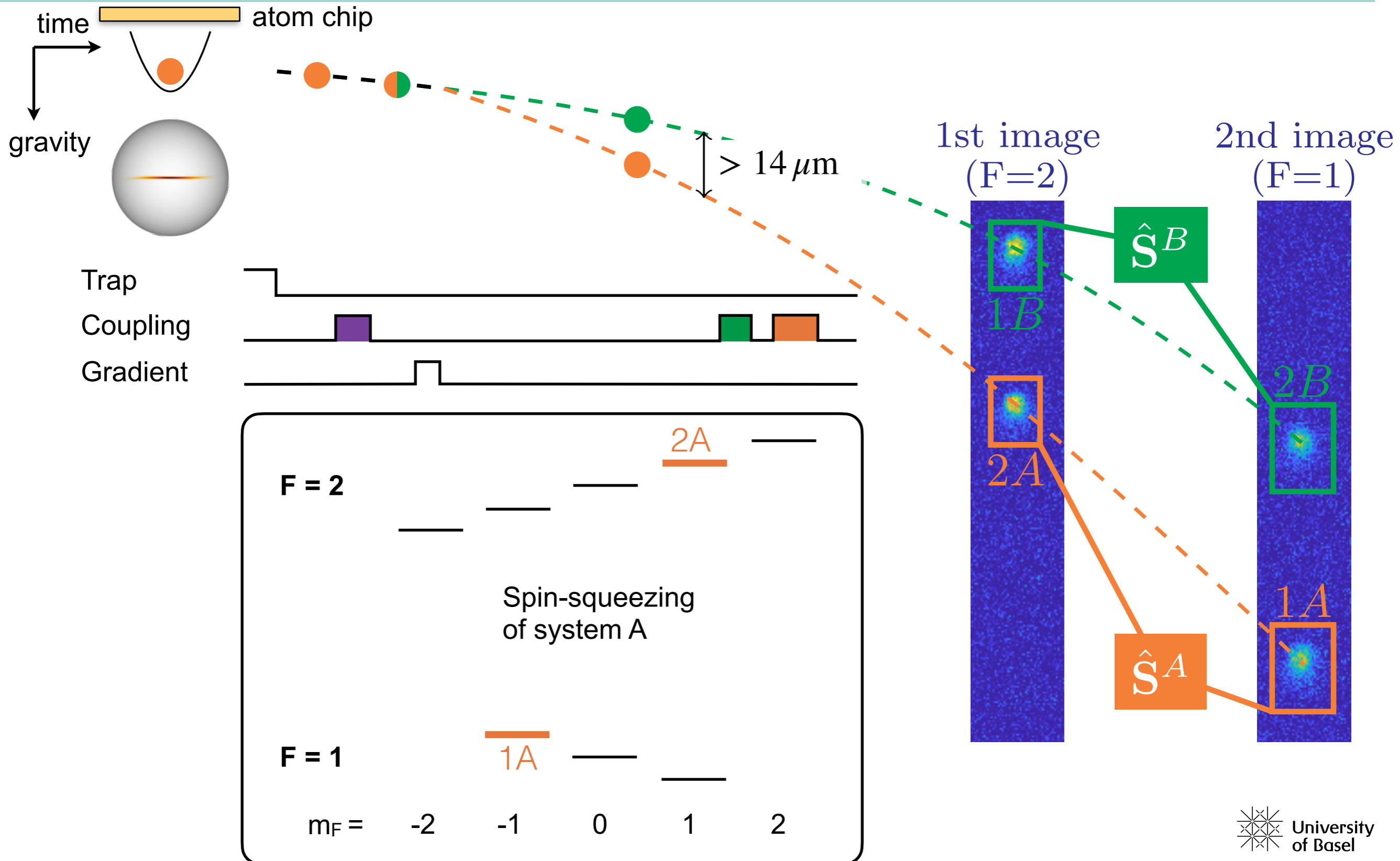
An EPR experiment with a many-particle system



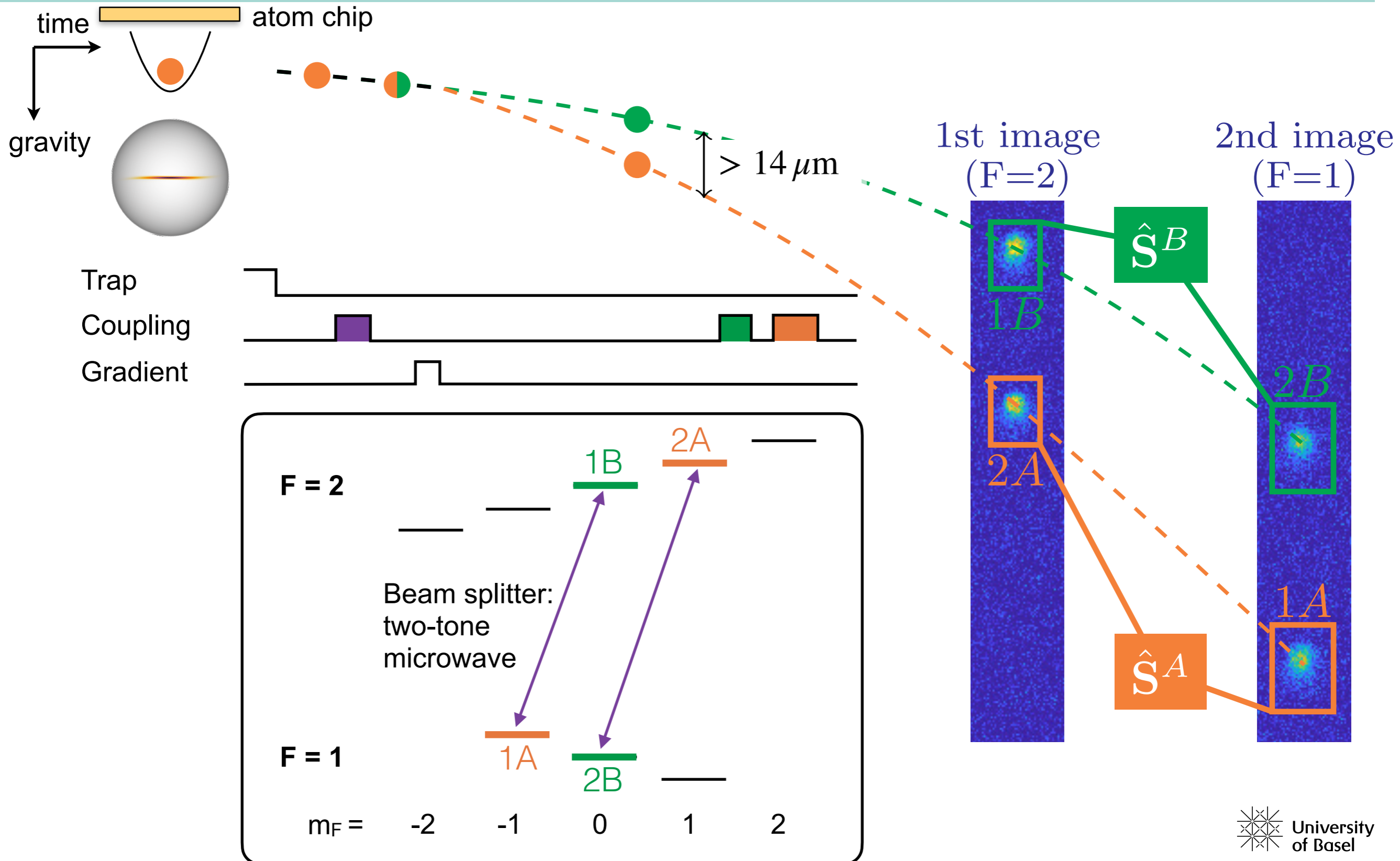
EPR, Phys Rev 47, 777 (1935)
Bohm, Quantum Theory (1951)

N. Killoran, M. Cramer, M. B. Plenio,
Extracting Entanglement from Identical Particles,
PRL 112, 150501 (2014)

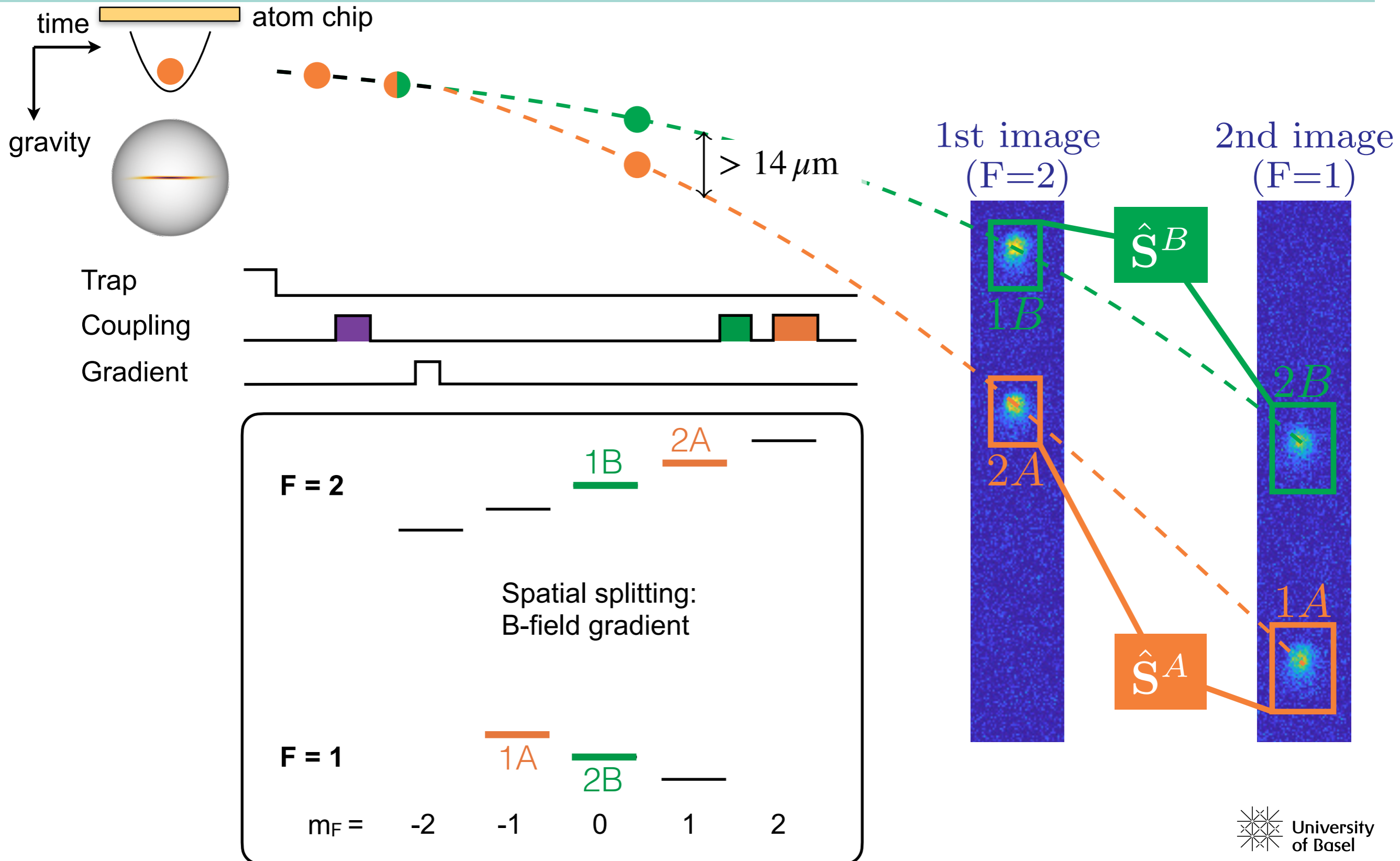
Coherent spatial splitting of two-component BEC



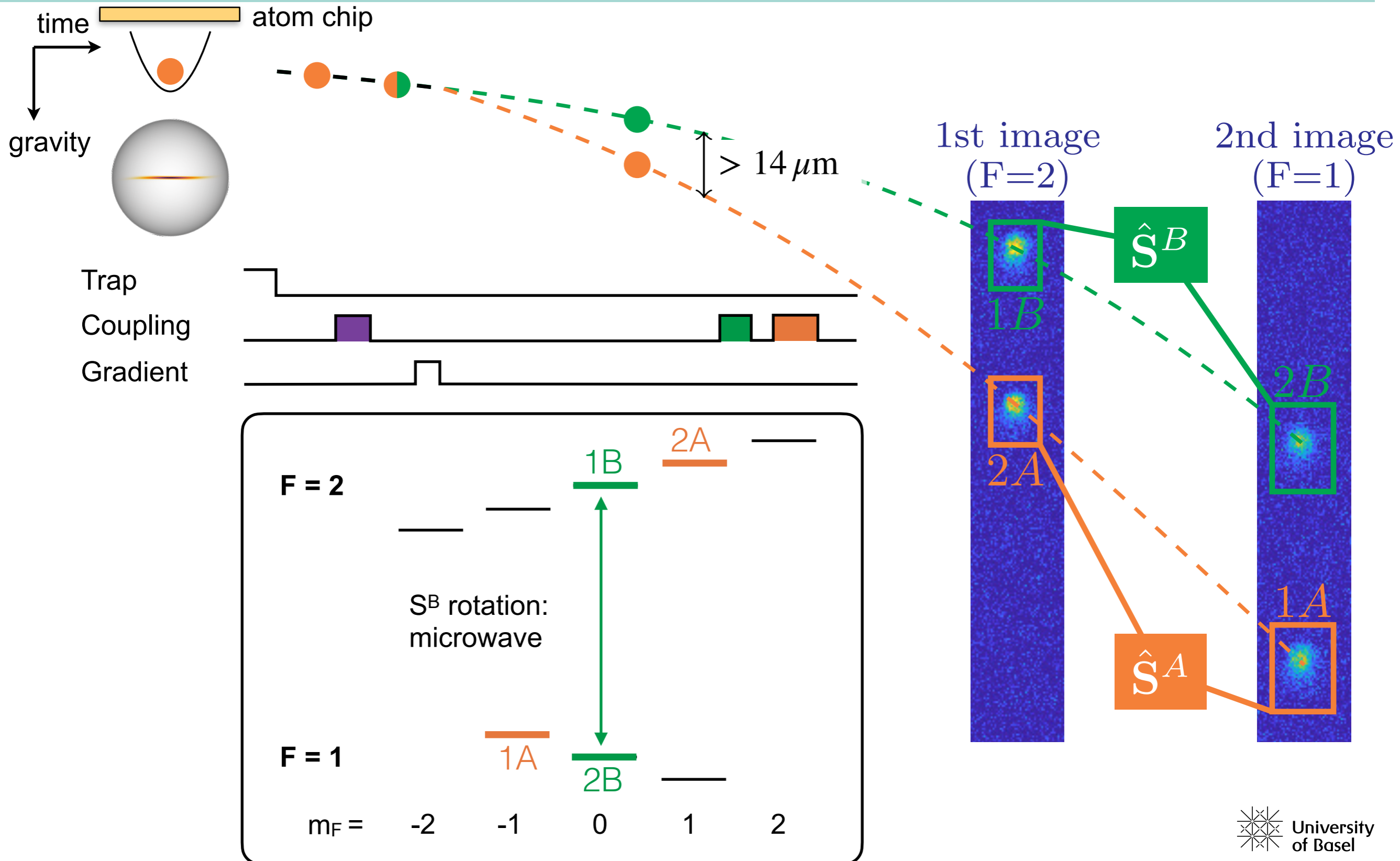
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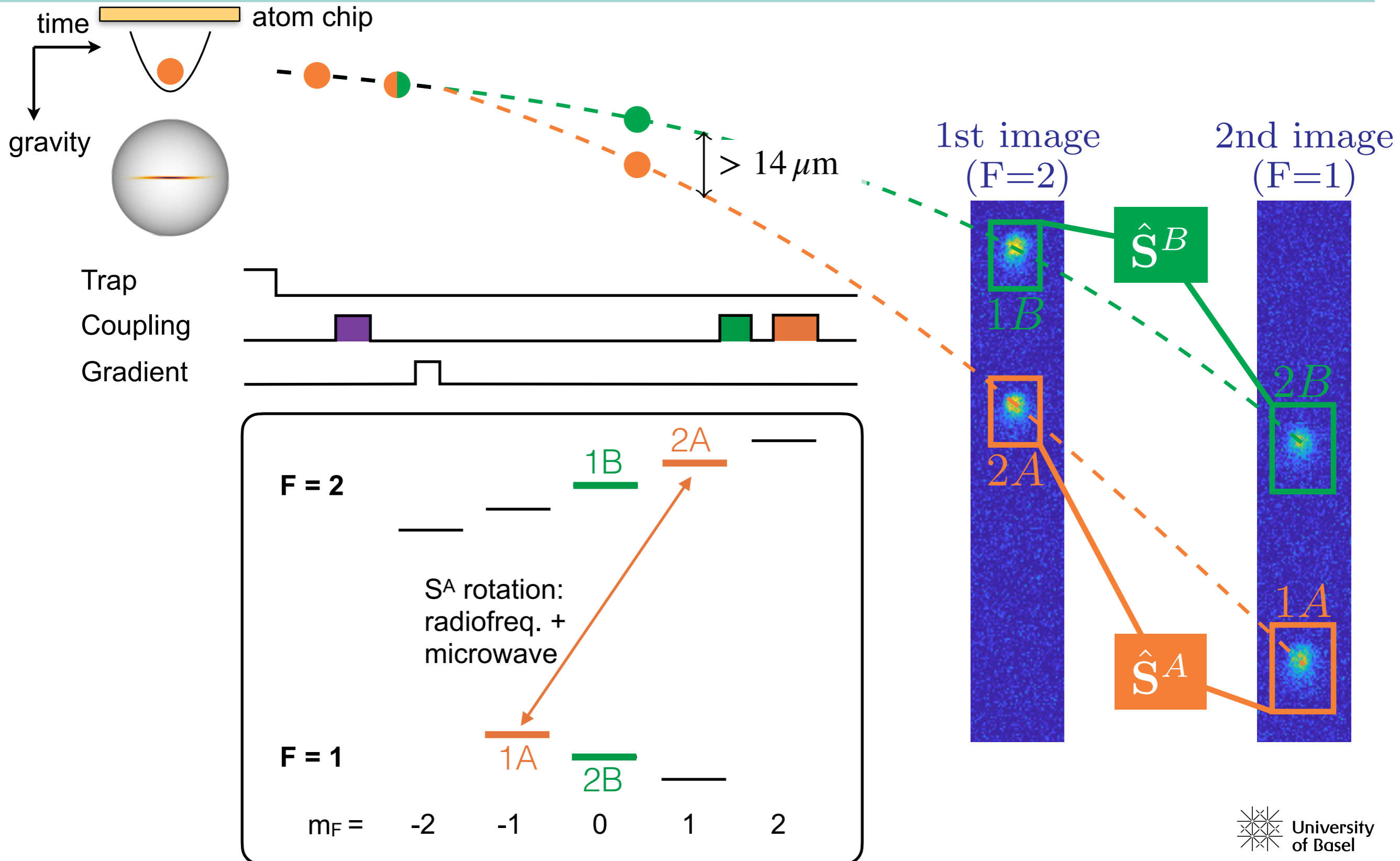
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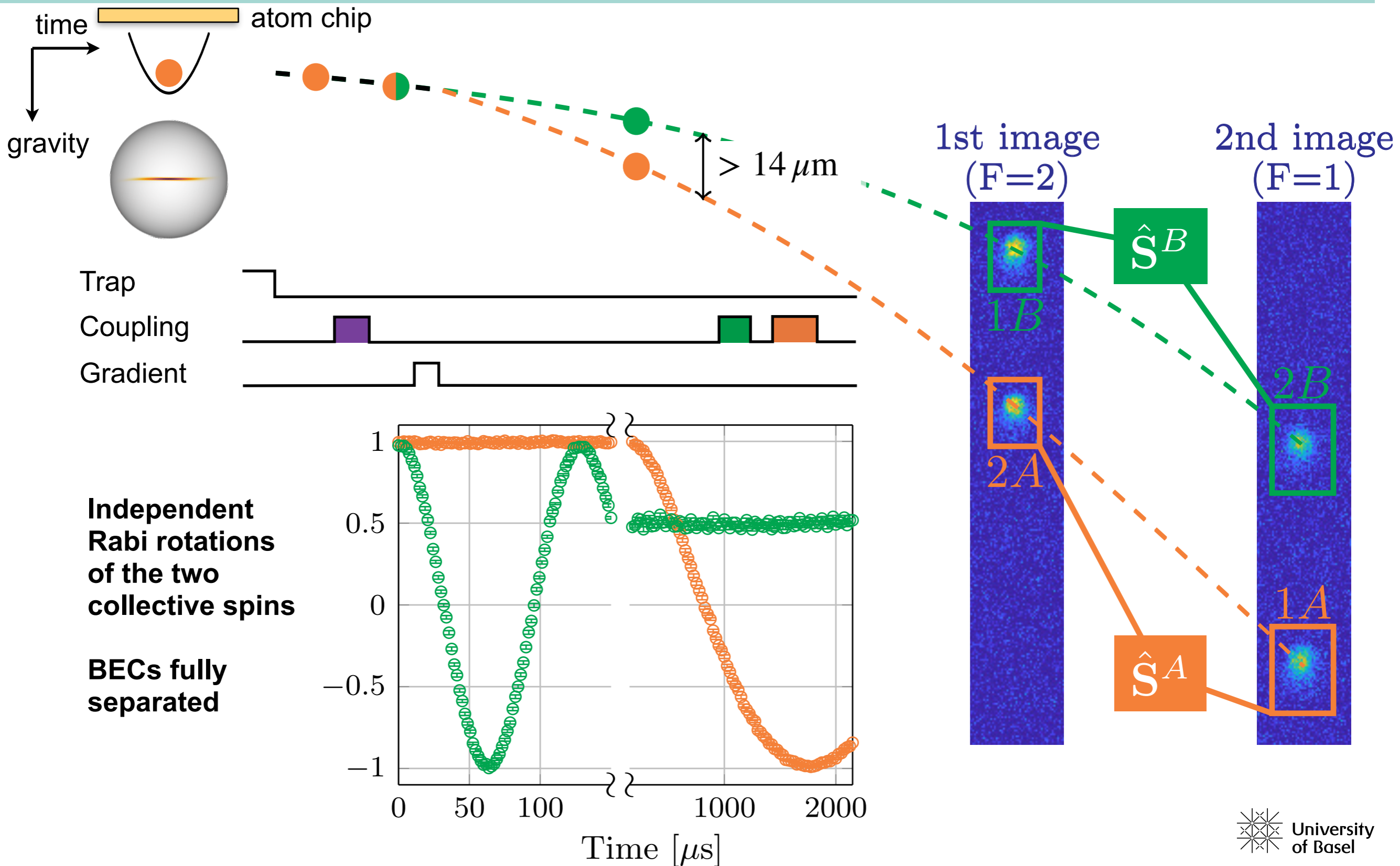
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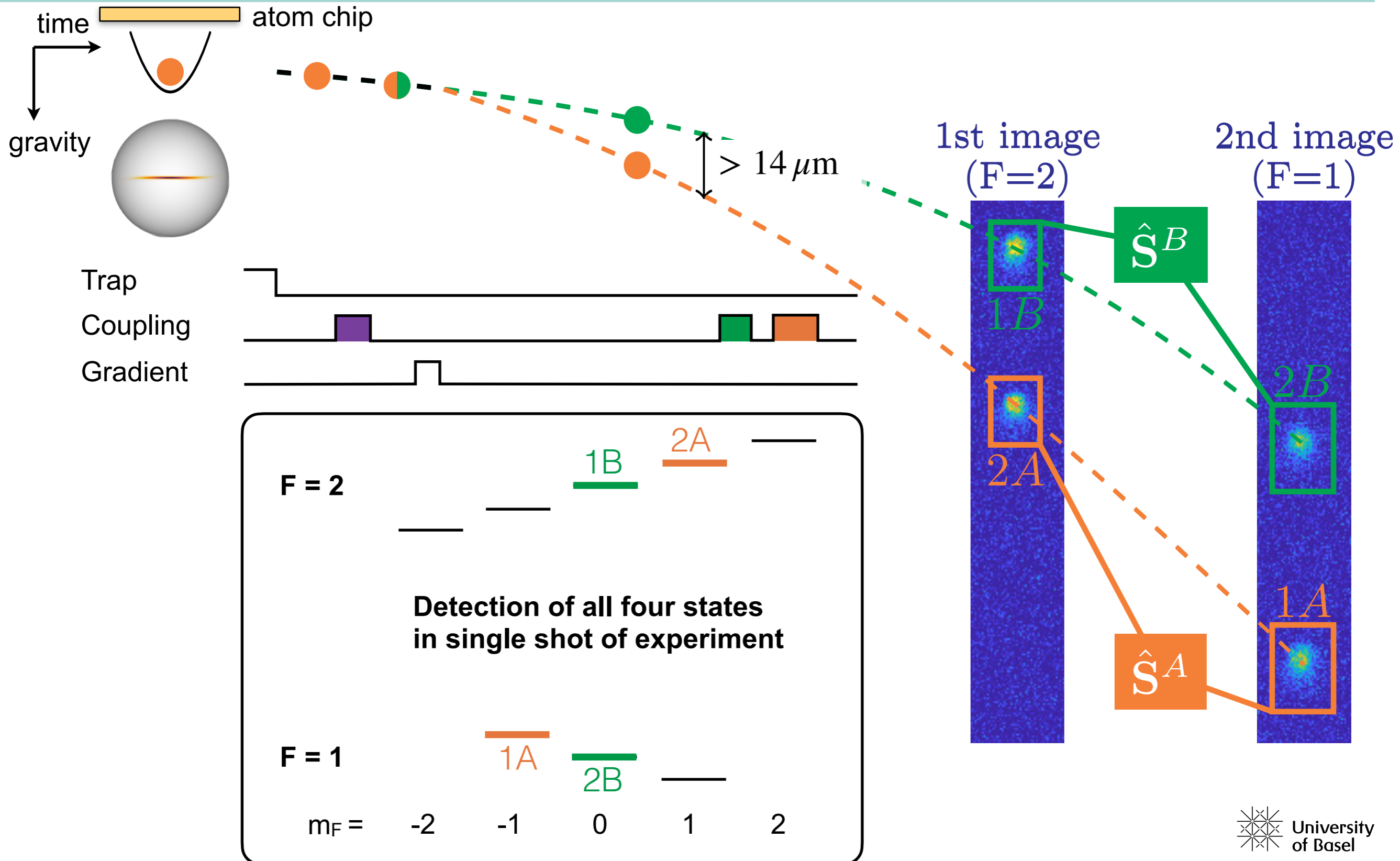
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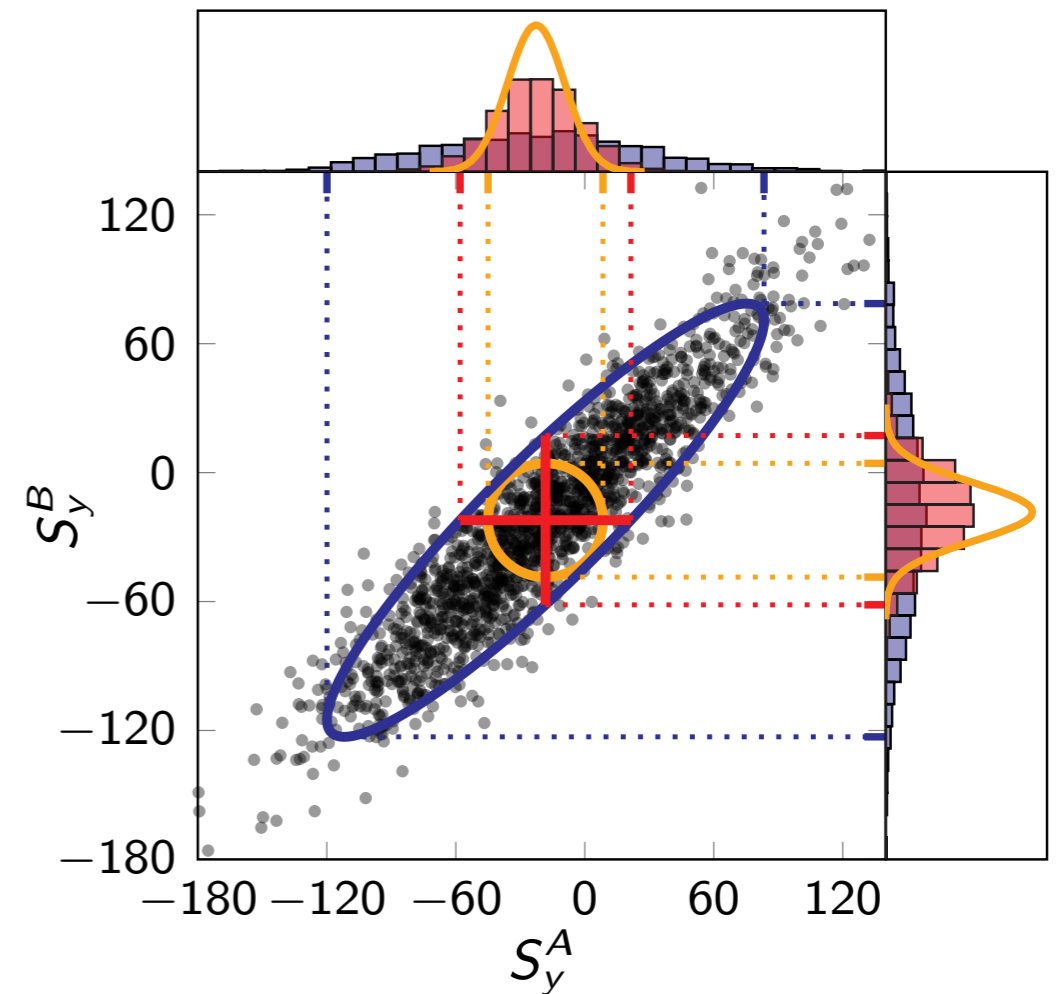
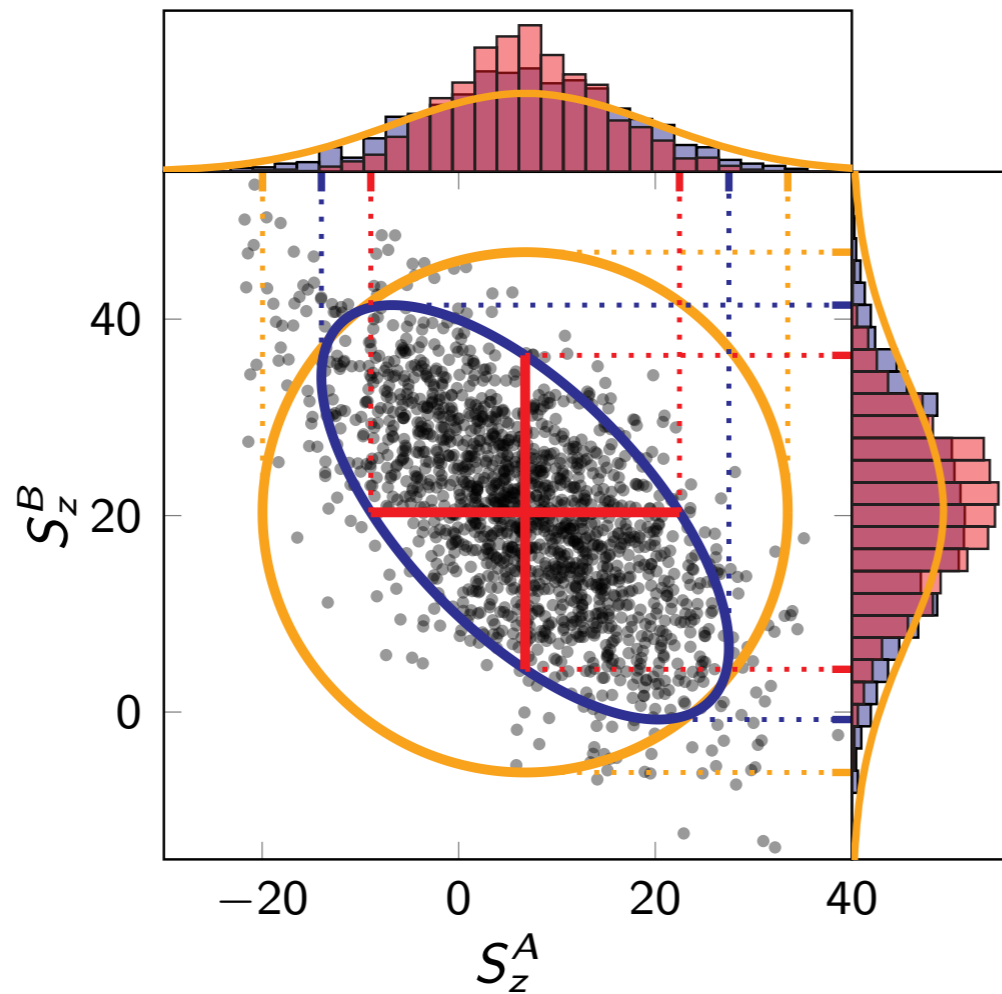
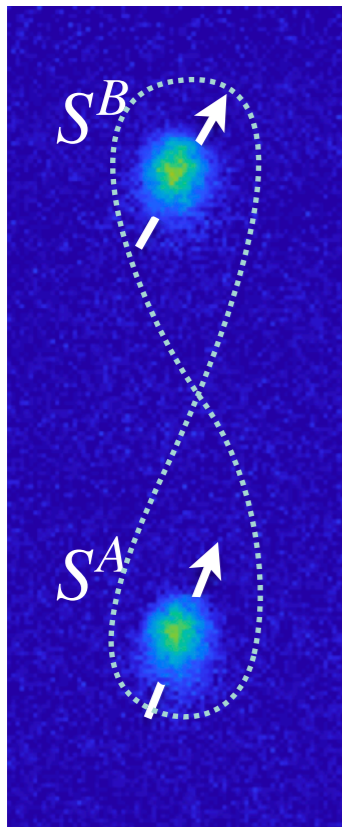
Independent spin rotations after splitting



Independent spin rotations after splitting



EPR paradox between two BECs

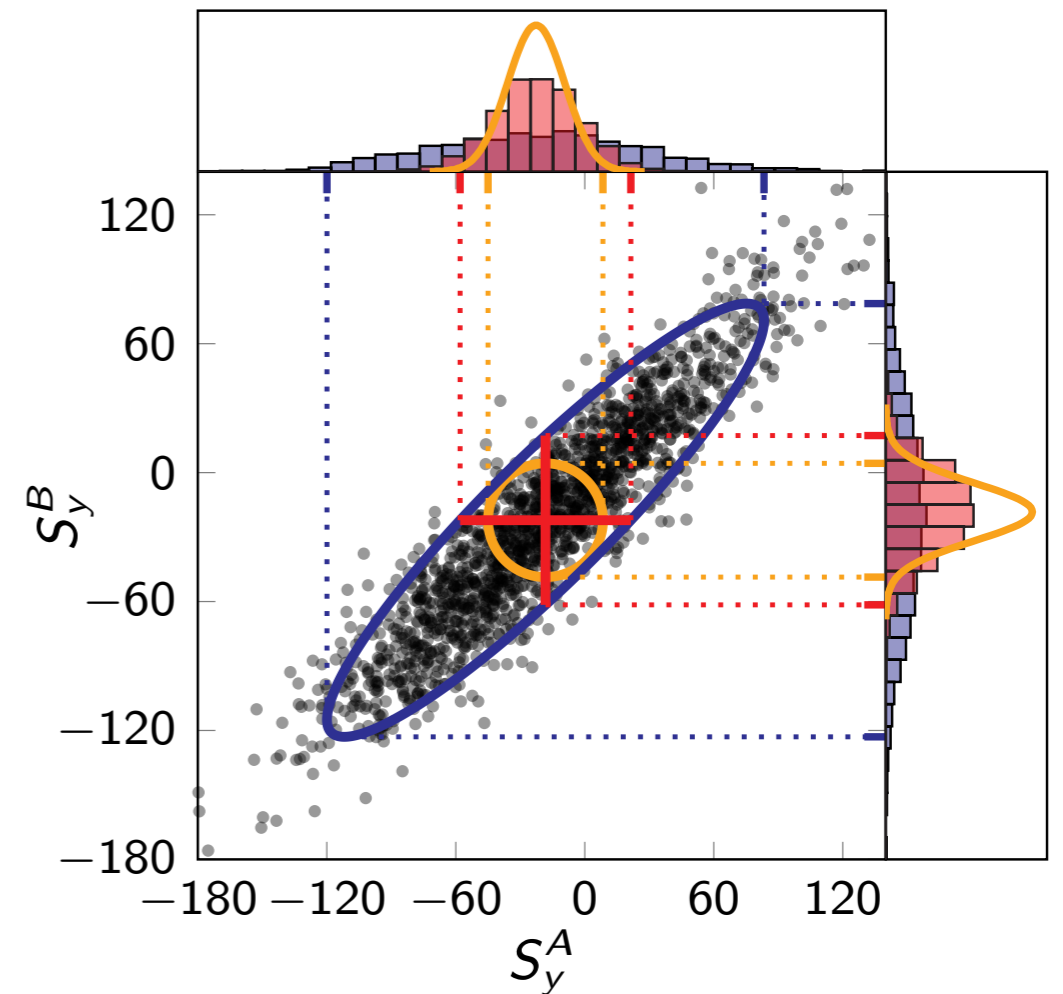
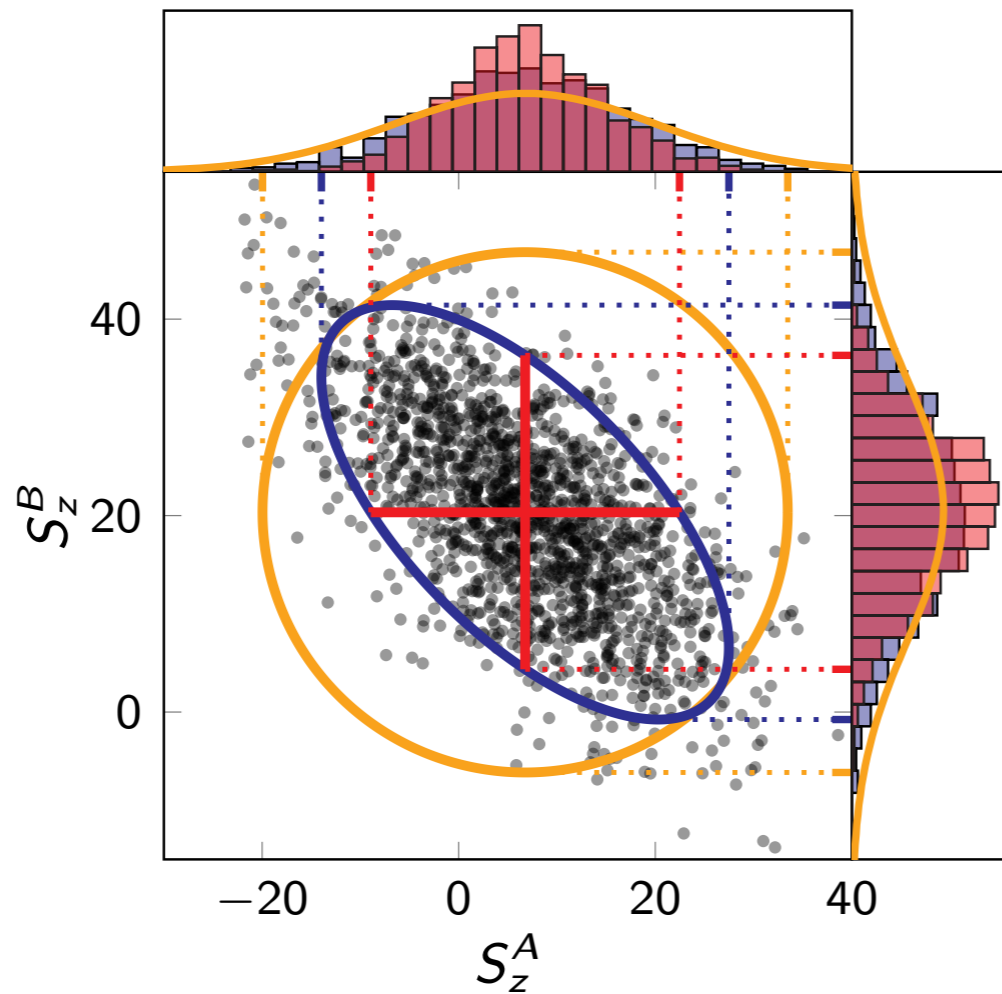
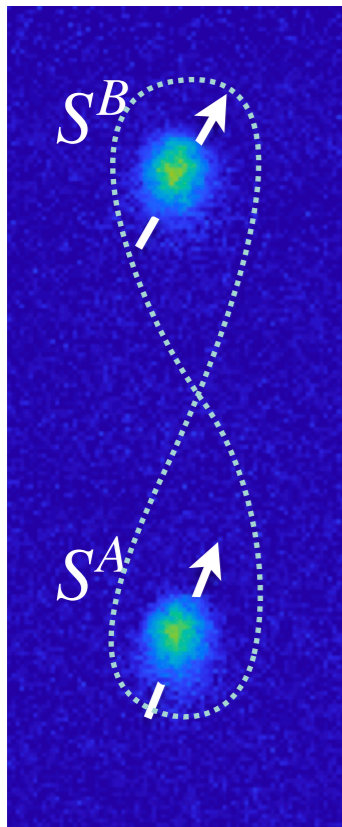


- $N \approx 1400$ atoms
- initial squeezing $\xi^2 \approx -7$ dB
- split spins are still squeezed
- contrast in S_x^A and S_x^B about 96%

EPR criterion

$$E_{EPR}^{A \rightarrow B} = \frac{4 \text{Var}(\hat{S}_y^B - g_y \hat{S}_y^A) \text{Var}(\hat{S}_z^B - g_z \hat{S}_z^A)}{|\langle \hat{S}_x^B \rangle|^2} = 0.81(3) < 1$$

Entanglement criterion

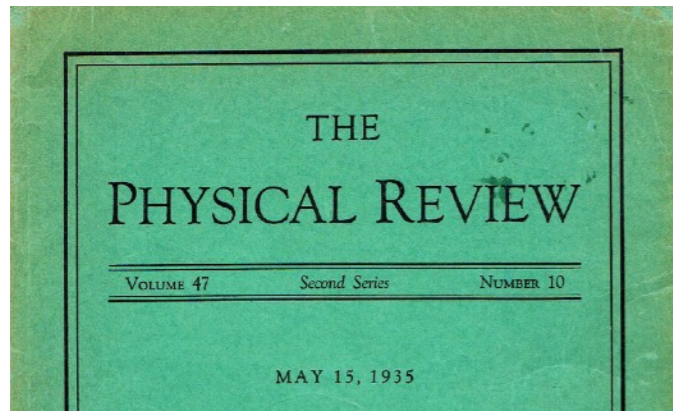


- $N \approx 1400$ atoms
- initial squeezing $\xi^2 \approx -7$ dB
- split spins are still squeezed
- contrast in S_x^A and S_x^B about 96%

Entanglement criterion

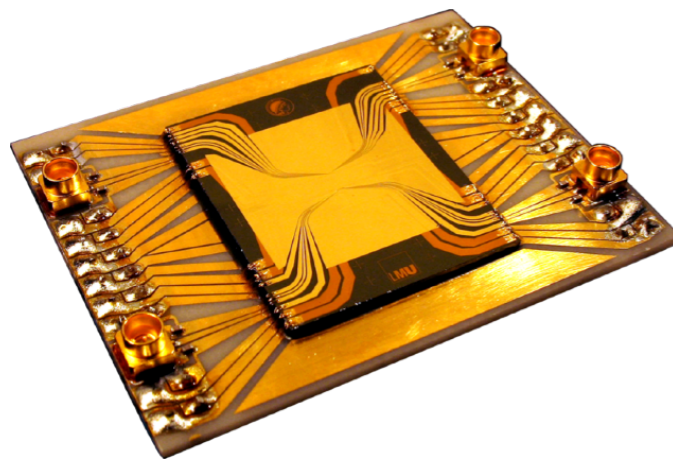
$$E_{Ent} = \frac{4 \text{Var}(\hat{S}_y^B - g_y \hat{S}_y^A) \text{Var}(\hat{S}_z^B - g_z \hat{S}_z^A)}{\left(|\langle \hat{S}_x^B \rangle| + |g_y g_z| |\langle \hat{S}_x^A \rangle|\right)^2} = 0.35(2) < 1$$

Outline



The Einstein-Podolsky-Rosen paradox

Einstein, Podolsky, Rosen, Phys Rev 47, 777 (1935)



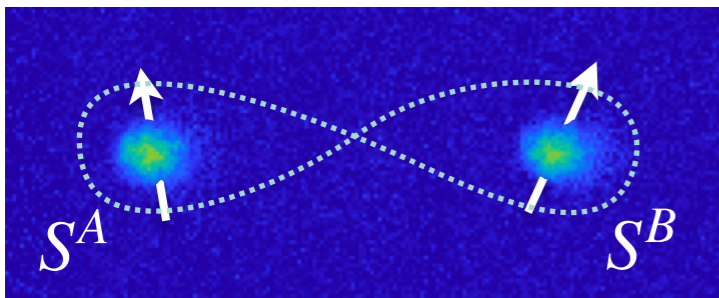
Two-component Rb BEC on atom chip Spin-squeezing, quantum metrology

Riedel et al, Nature 464, 1170 (2010)

Ockeloen et al, PRL 111, 143001 (2013)

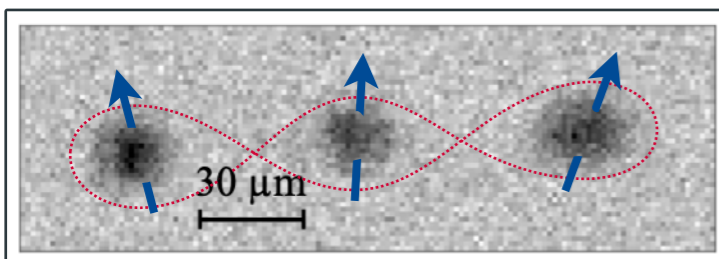
Schmied et al, Science 352, 441 (2016)

Fadel et al, Science 360, 409 (2018)



EPR paradox between two spatially separated and addressable BECs

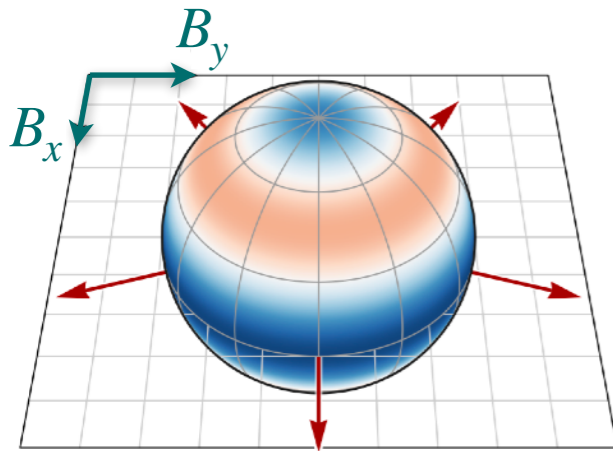
Colciaghi et al, PRX 13, 021031 (2023)



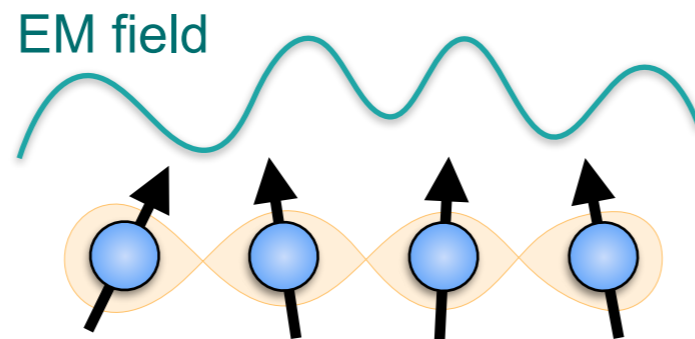
Multiparameter estimation with an array of entangled atomic sensors

collaboration: Y. Baamara, A. Sinatra

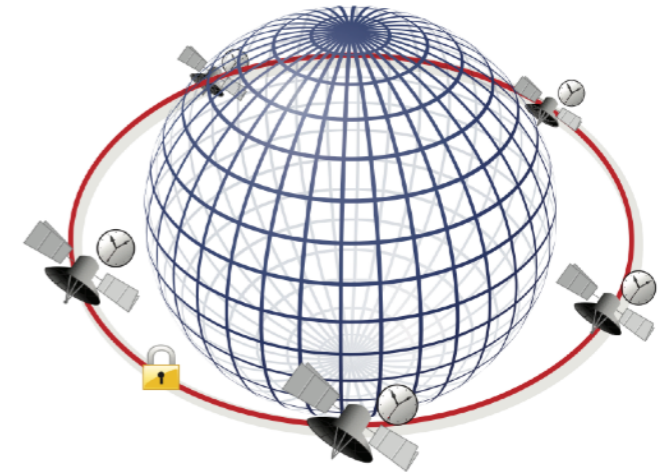
Multiparameter quantum metrology



Quantum compass
Vasilyev et al, arXiv:2404.14194



Sensor array for field imaging
Baamara et al, Scipost Phys 14, 050 (2023)



Quantum network of clocks
Kómár et al, Nat Phys 10, 582 (2014)

Multiparameter sensing tasks:

- vector field sensing
- sensing field distributions
- imaging
- networks of clocks/sensors

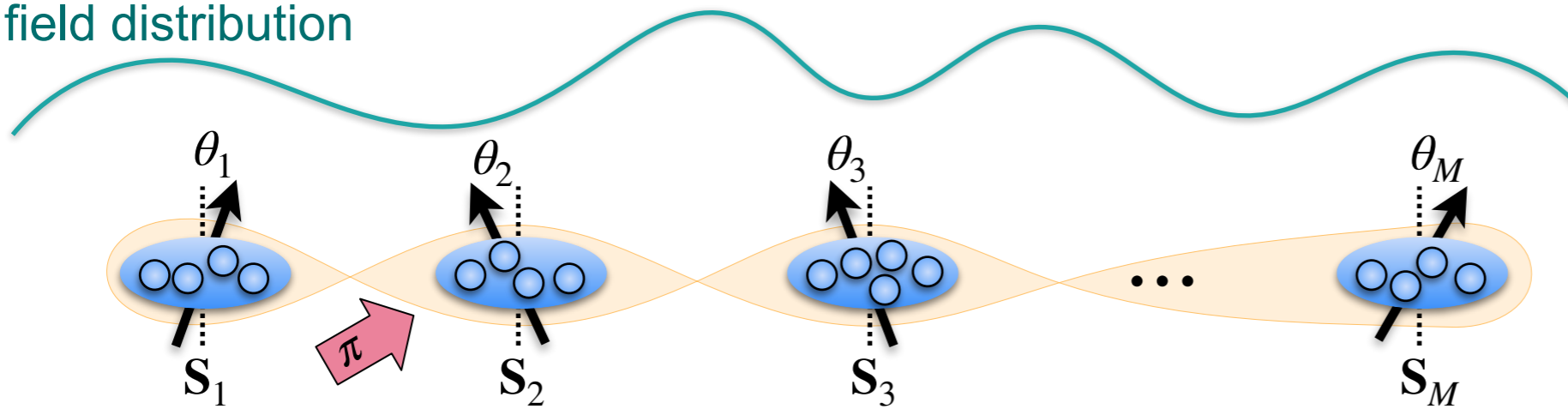
Conceptual challenges:

- measurement incompatibility $[S_i, S_j] = i\epsilon_{ijk}S_k$
- intra-sensor vs. inter-sensor entanglement
- quantum gain depends on resources considered (local vs. global detection, ...)
- optimal strategy for given resources?

Szczykulska et al, Adv Phys: X, 1, 621 (2016); Gessner et al, PRL 121, 130503 (2018); Proctor et al, PRL 120, 080501 (2018); Sidhu et al, AVS Quantum Sci 2, 014701 (2020); Demkowicz-Dobrzański et al, J Phys A: Math Theor 53, 363001 (2020); Albarelli et al, Phys Lett A, 384, 126311 (2020), ...

Multiparameter sensing with distributed entanglement

field distribution



array of entangled atomic ensembles

- fixed total N
- fixed total number of preparations μ

Global squeezing of all ensembles: $\mathbf{S} = \sum_k \mathbf{S}_k$ squeezed with $\xi < 1$

→ only symmetric mode has reduced noise

$$\theta_{\text{sym}} = (\theta_1 + \theta_2 + \dots + \theta_M) / \sqrt{M} \quad \Delta\theta_{\text{sym}} = \xi \Delta\theta_{\text{SQL}} \quad \Delta\theta_{\text{SQL}} = \sqrt{\frac{M}{\mu N}}$$

Local spin rotations (π -pulses) transfer quantum enhancement to target Hadamard mode

e.g. $\theta_{\text{target}} = (\theta_1 - \theta_2 + \dots + \theta_M) / \sqrt{M}$

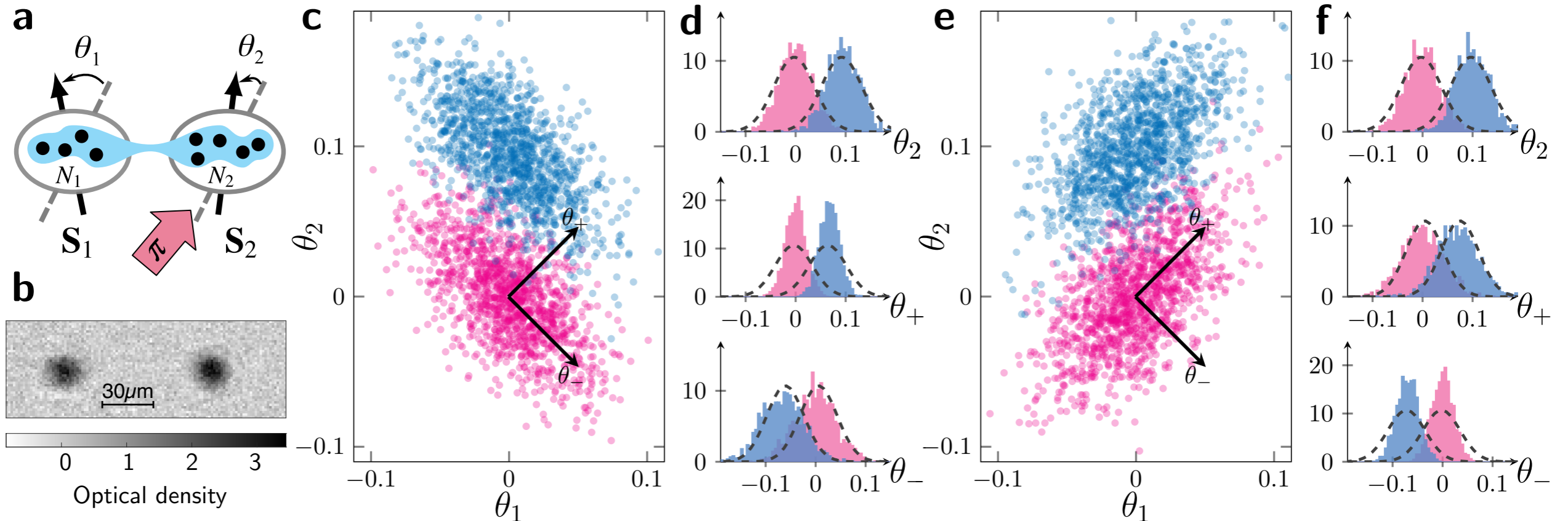
$$\Delta\theta_{\text{target}} = \xi \Delta\theta_{\text{SQL}}$$

Prepare & measure complete set of target modes

→ all θ_i quantum enhanced
(optimal strategy saturating CR bound)

$$\Delta\theta_i = \frac{\sqrt{M}\xi}{\sqrt{1 + (M-1)C^2\xi^2}} \Delta\theta_{\text{SQL}}$$

Two clouds: squeezing common and differential mode



exploit individual coherent control and detection of sensors

common mode:

$$\theta_+ = (\theta_1 + \theta_2)/\sqrt{2}$$

squeezed by -5.6(2) dB

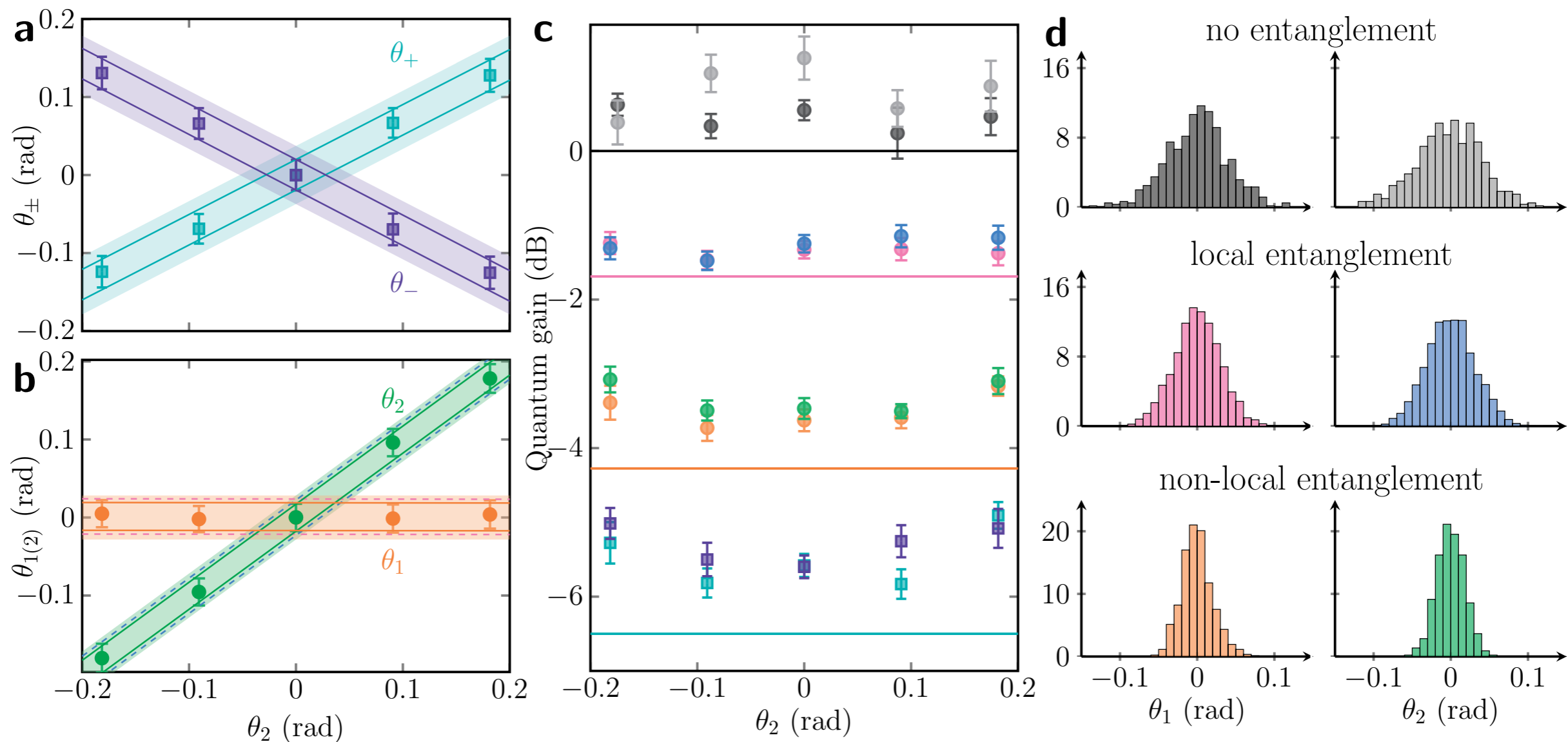
differential mode:

$$\theta_- = (\theta_1 - \theta_2)/\sqrt{2}$$

squeezed by -5.6(2) dB

see also: Malia et al,
Nature 612, 661 (2022)

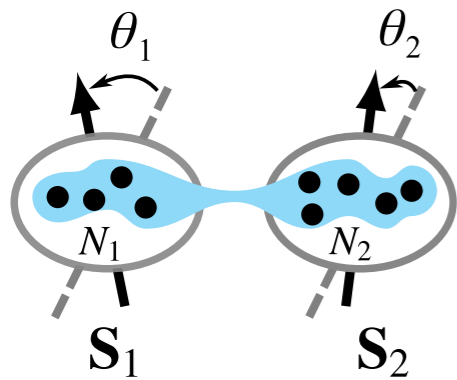
Joint quantum enhancement for both parameters



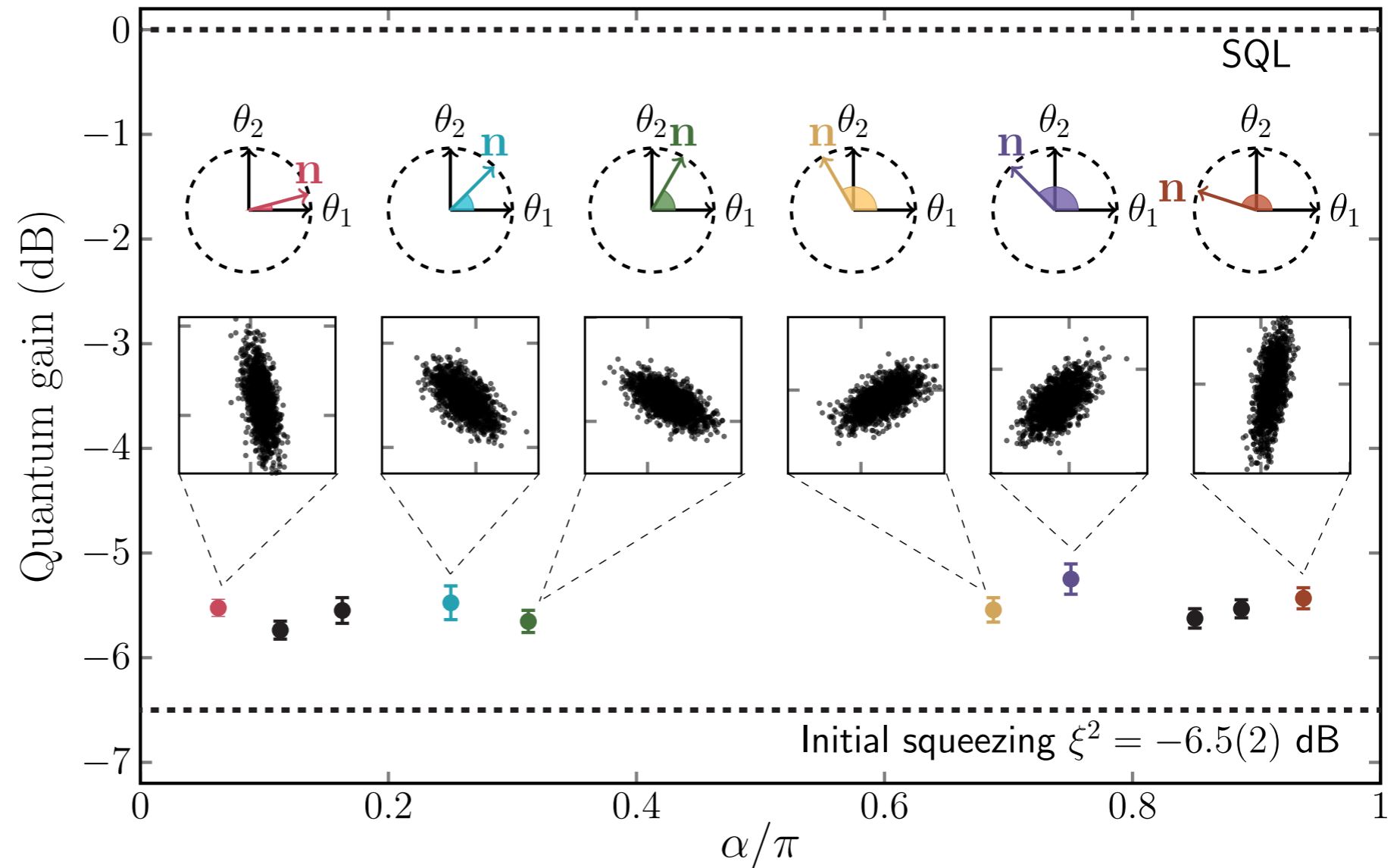
Distributed entanglement results in **joint** quantum enhancement of **both** parameter estimates by 3.6(2) dB

$$\frac{(\Delta\theta_i)^2}{(\Delta\theta_{\text{SQL}})^2} = \frac{2\xi^2}{1 + C^2\xi^2}$$

Squeezing arbitrary nonlocal parameter combinations

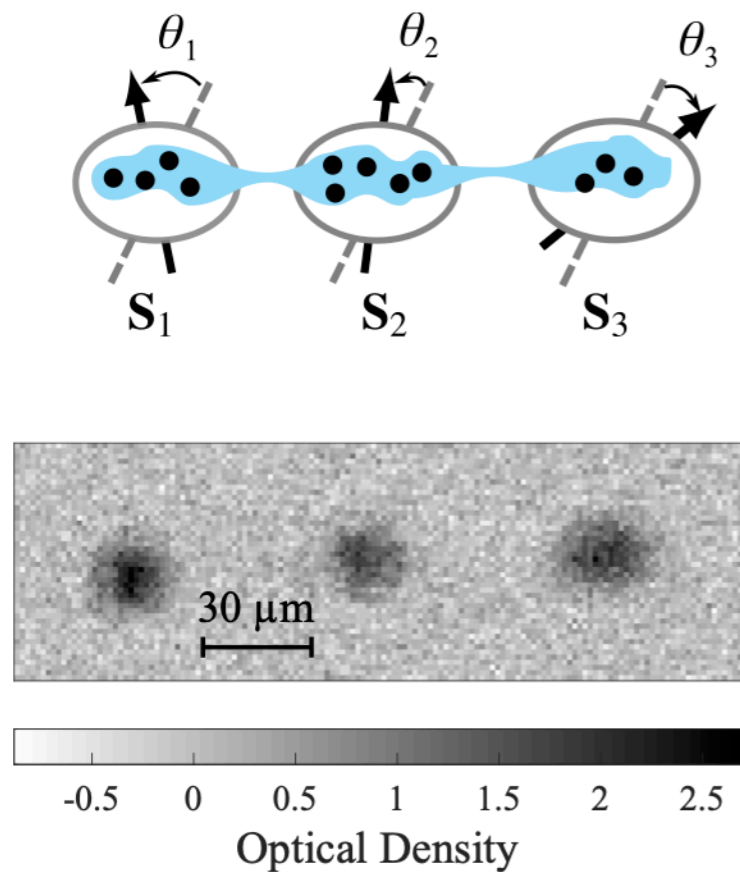


adjust N_1 and N_2 to match parameter combination

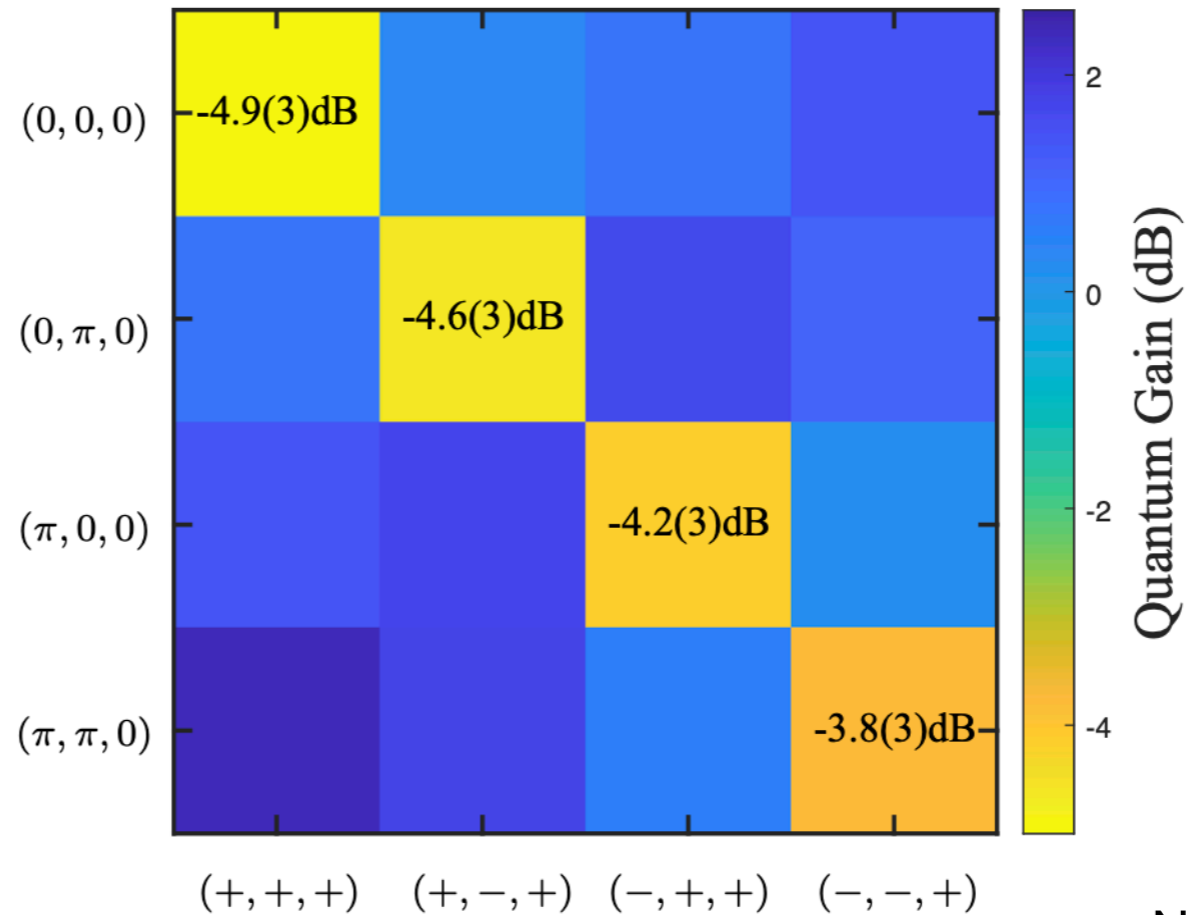


nonlocal parameter combinations: $\mathbf{n} \cdot \boldsymbol{\theta} = \cos(\alpha) \theta_1 + \sin(\alpha) \theta_2$

Distributed sensing with three entangled ensembles



preparation of three entangled spinor BECs

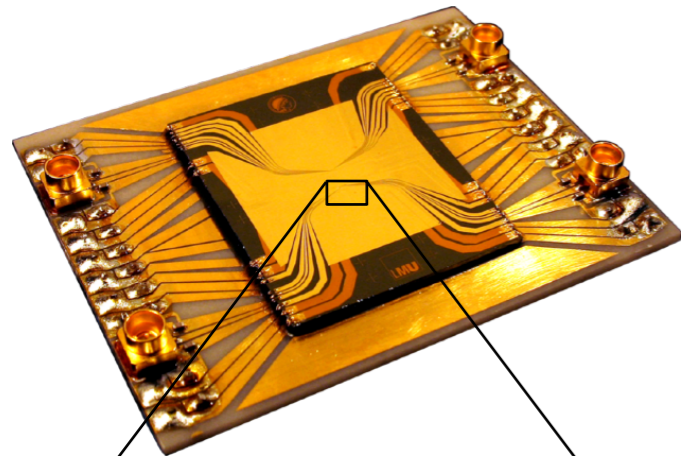


quantum enhancement of four different modes

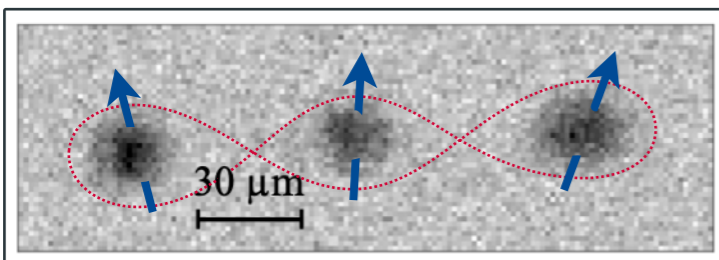
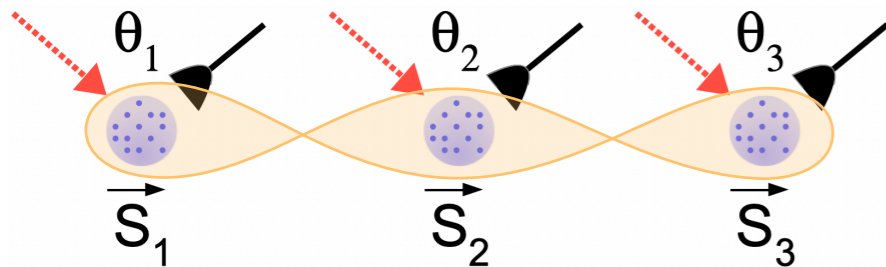
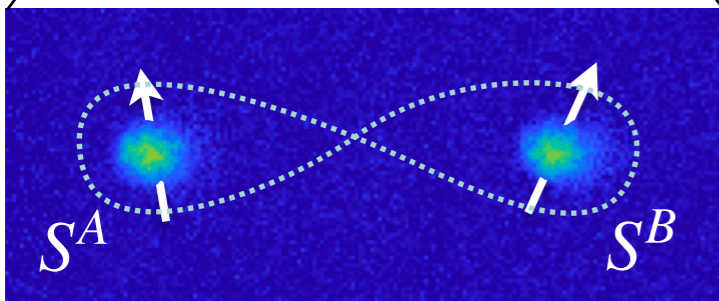
No Hadamard matrix for $M = 3 \rightarrow$ measure four modes for optimal estimation of all θ_i

Joint quantum gain for multiple parameters from distributed entanglement

Outlook



$$E_{EPR}^{B \rightarrow A} = 0.77(3) < 1$$



Experiments on quantum foundations with massive many-particle systems

- Bell test with spatially separated BECs?
→ non-Gaussian states or measurements
[Oudot et al, New J Phys 21, 103043 \(2019\)](#)
- Macroscopicity of many-particle EPR?
[Leggett, J Phys A 40, 3141 \(2007\)](#)
[Cavalcanti and Reid, J Mod Opt 54, 2373 \(2007\)](#)
- Entanglement in tripartite systems

Multiparameter quantum metrology

- Imaging of field distributions close to chip
- Quantum gain for compressed sensing
[Baamara et al, Scipost Phys 14, 050 \(2023\)](#)
- Networks of clocks



University of Basel

Quantum optics and atomic physics

Positions available!



Manel Bosch



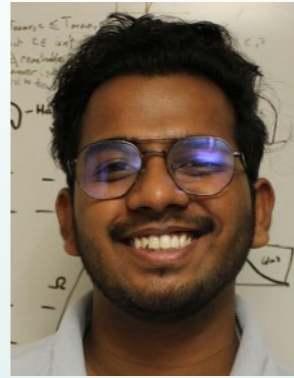
Gianni Buser



Paolo Colciaghi



Maryse Ernzer



Suyash Gaiwad



Alexandre Huot



Lex Joosten



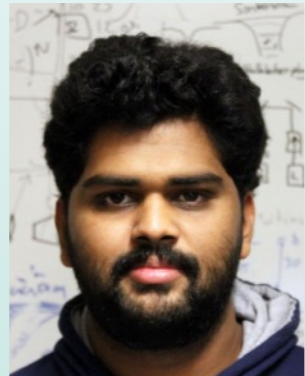
Yifan Li



Roberto Mottola



Haroon Saeed



Madhav Saravanan



Gian-Luca Schmid



Tilman Zibold

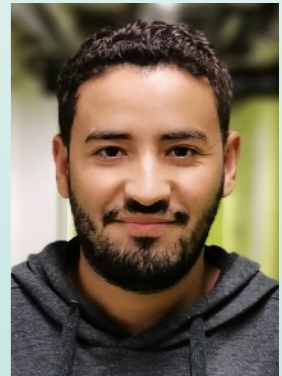


Philipp Treutlein

theory collaborators



Alice Sinatra



Youcef Baamara

