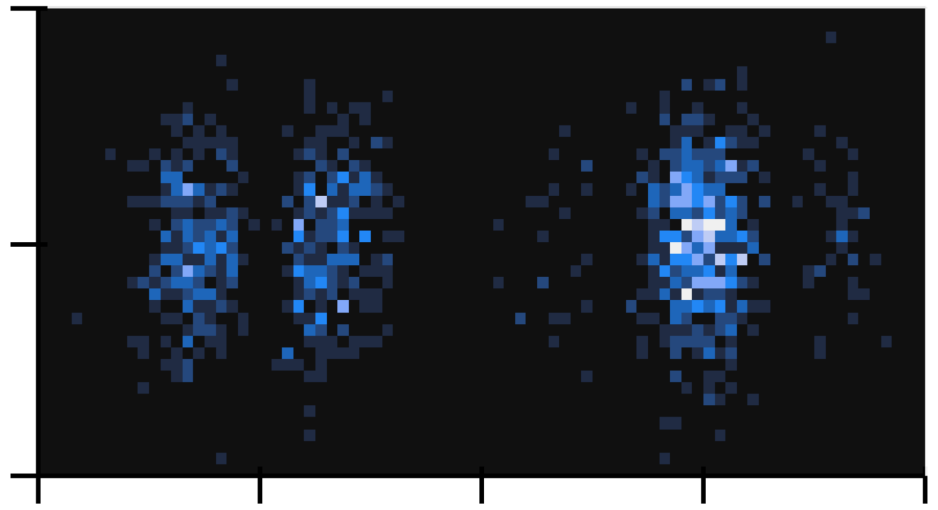
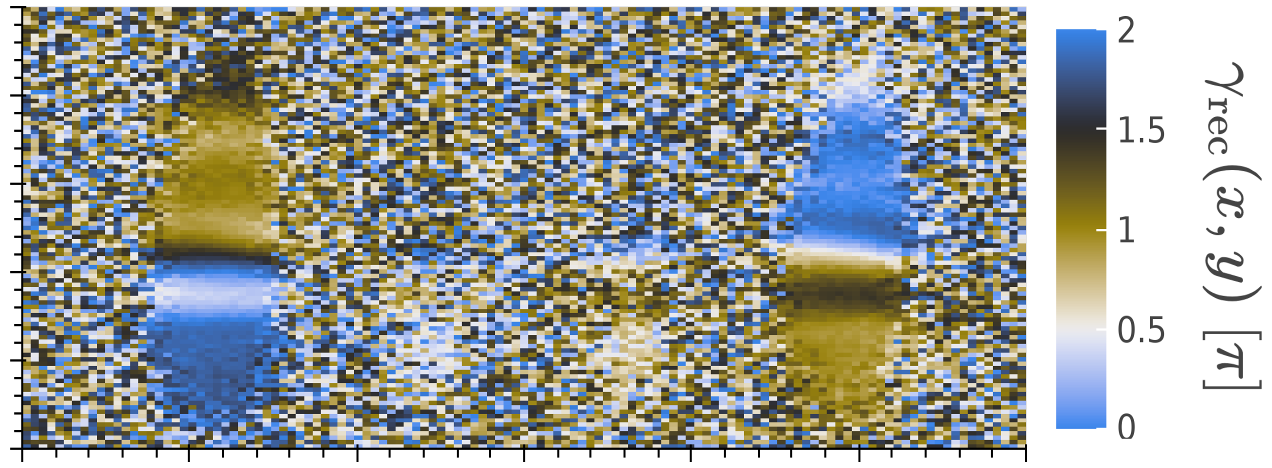
Theory
of Quantum Sensing

Leibniz
Universität
Hannover

Spatially Resolved Phase Reconstruction for Atom Interferometry

Jan-Niclas Kirsten-Siemß





arXiv: 2405.05150 (2024)

Stefan Seckmeyer, Jan-Niclas Kirsten-Siemß, Naceur Gaaloul
Experimental Collaboration: Holger Ahlers[1], Sven Abend, Matthias Gersemann, Sven Abend, Ernst M. Rasel
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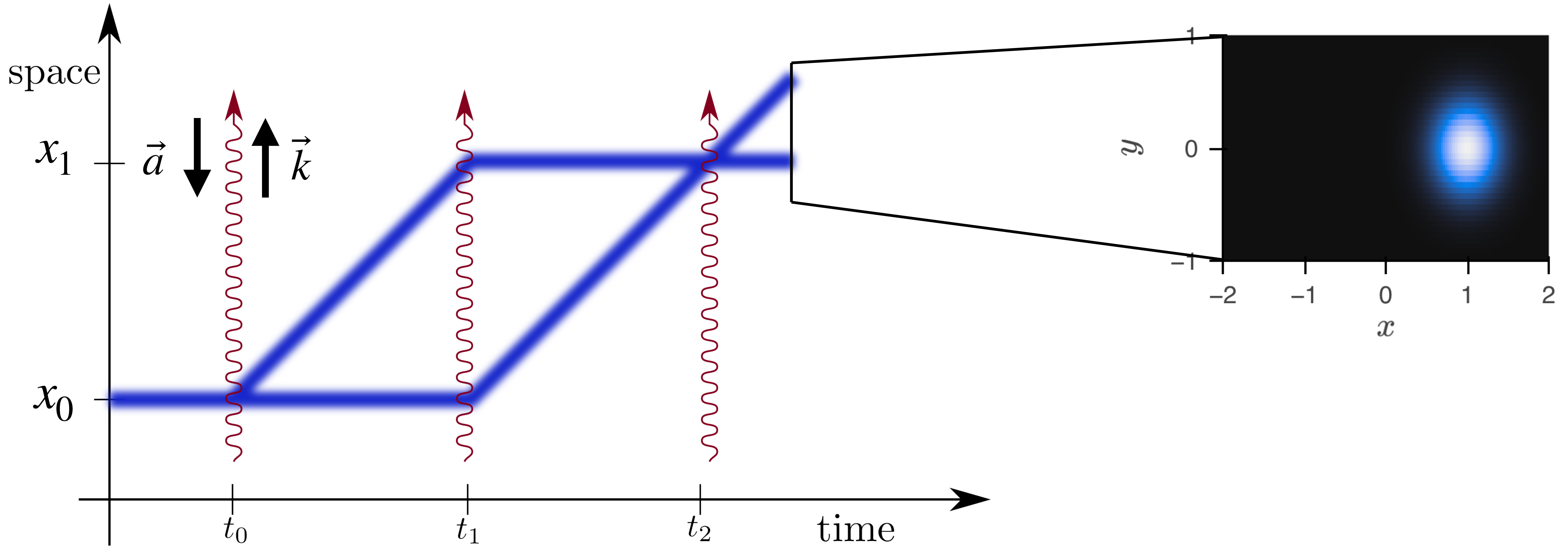
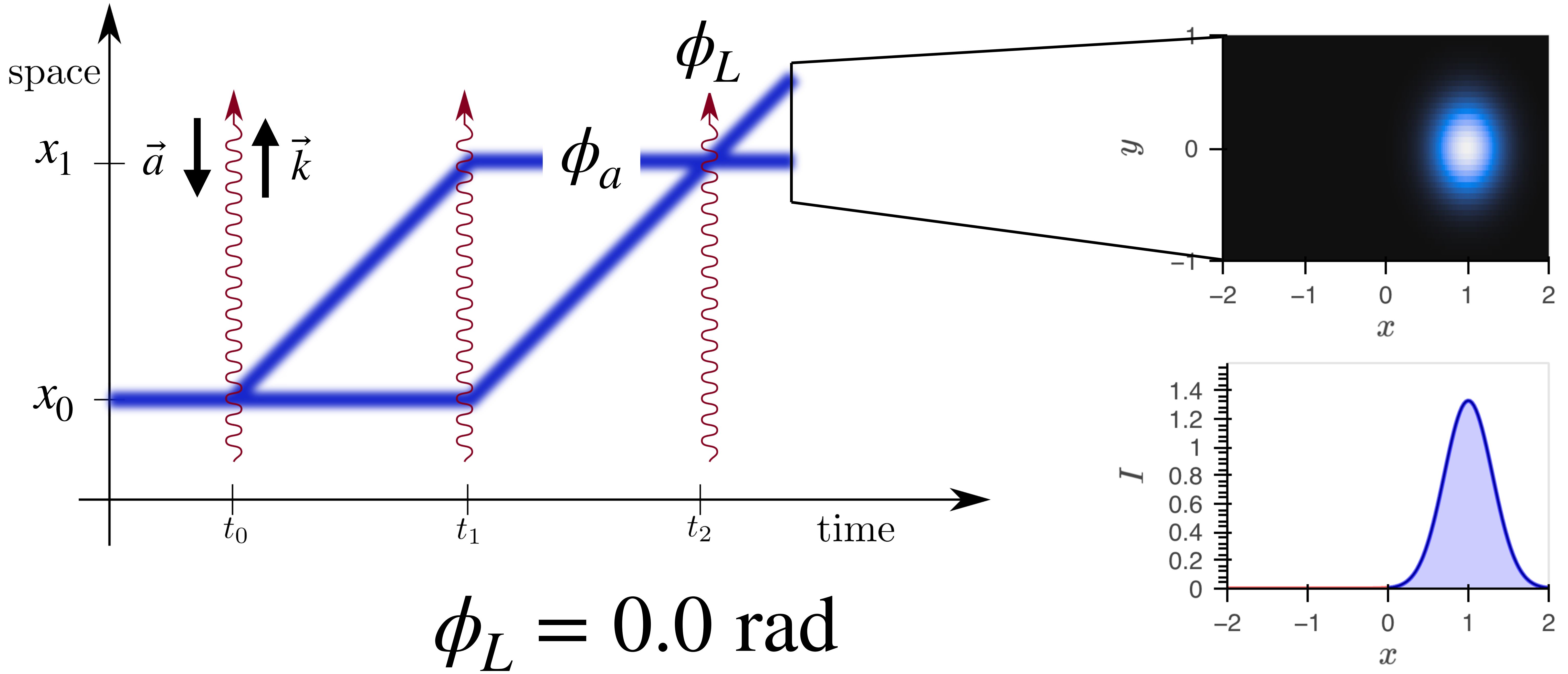
[1]Present Address: DLR Institute for Satellite Geodesy and Inertial Sensing, Hannover, 30167, Niedersachsen, Germany

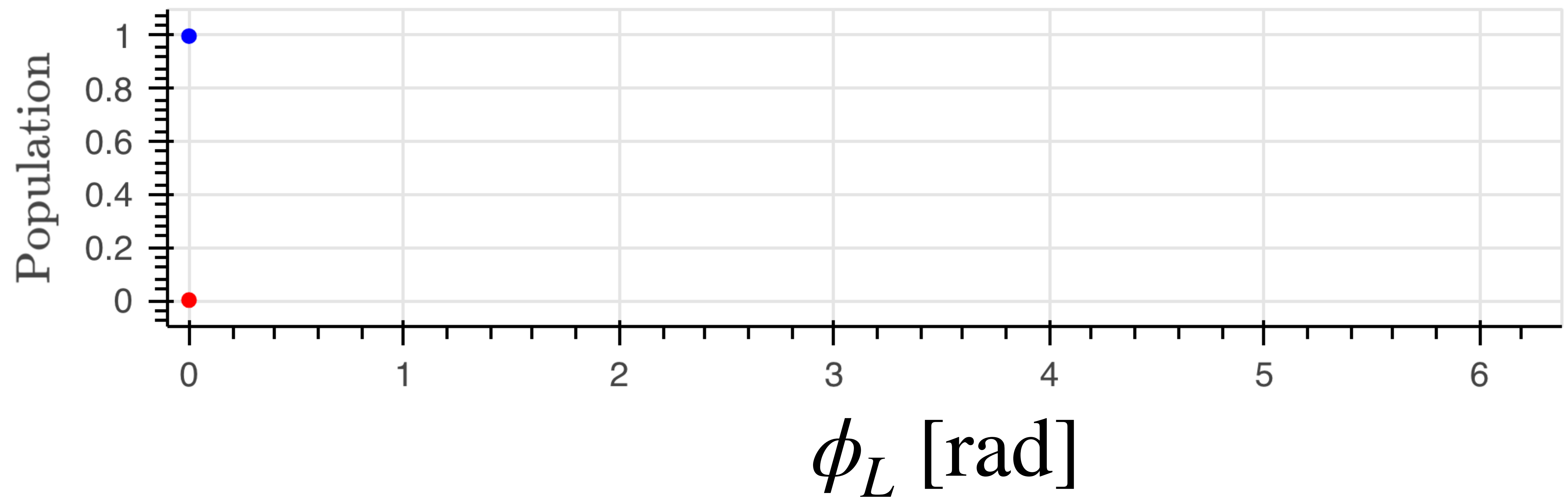
Mach-Zehnder Interferometer



Signal Extraction via Phase Scan



$\phi_L = 0.0$ rad

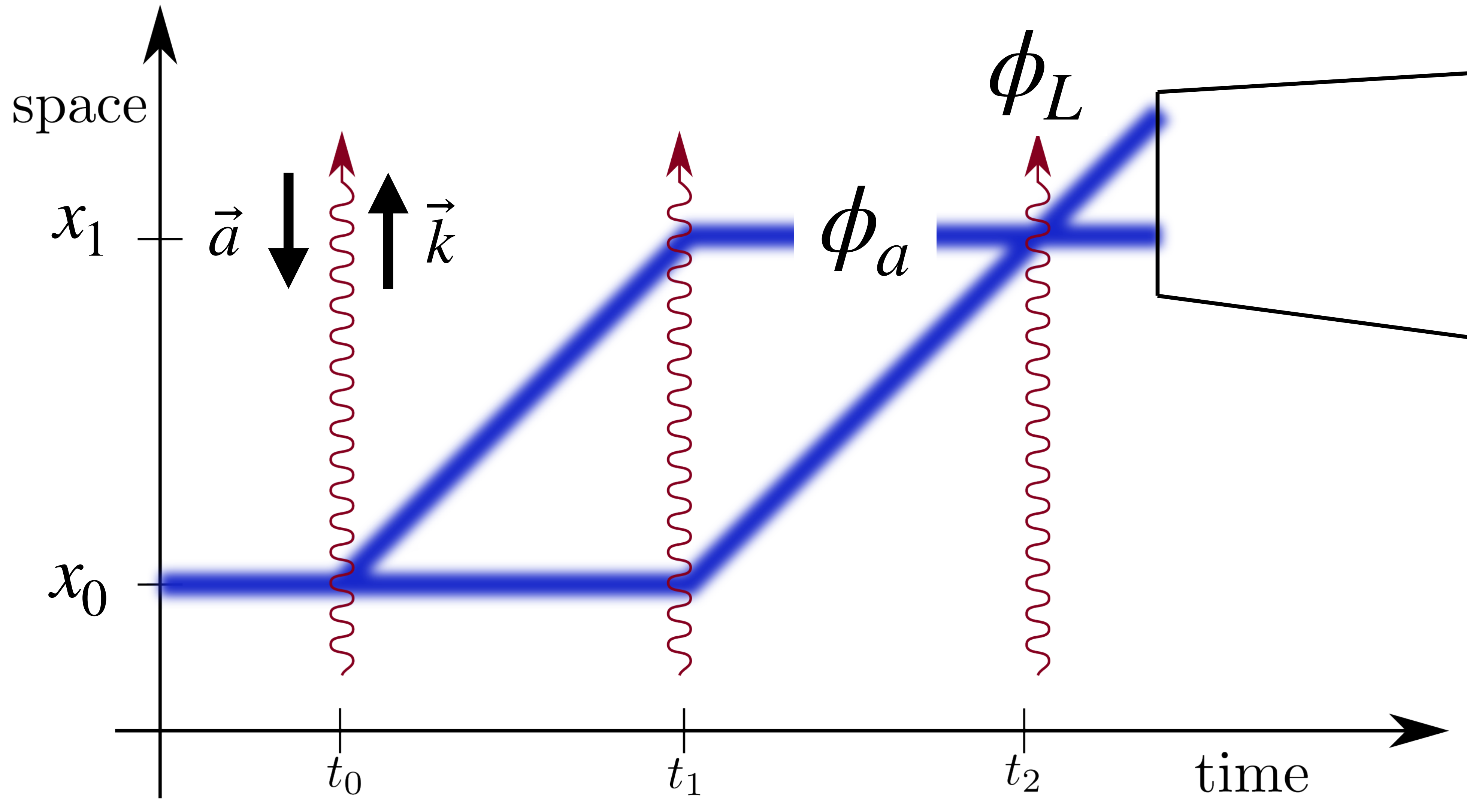


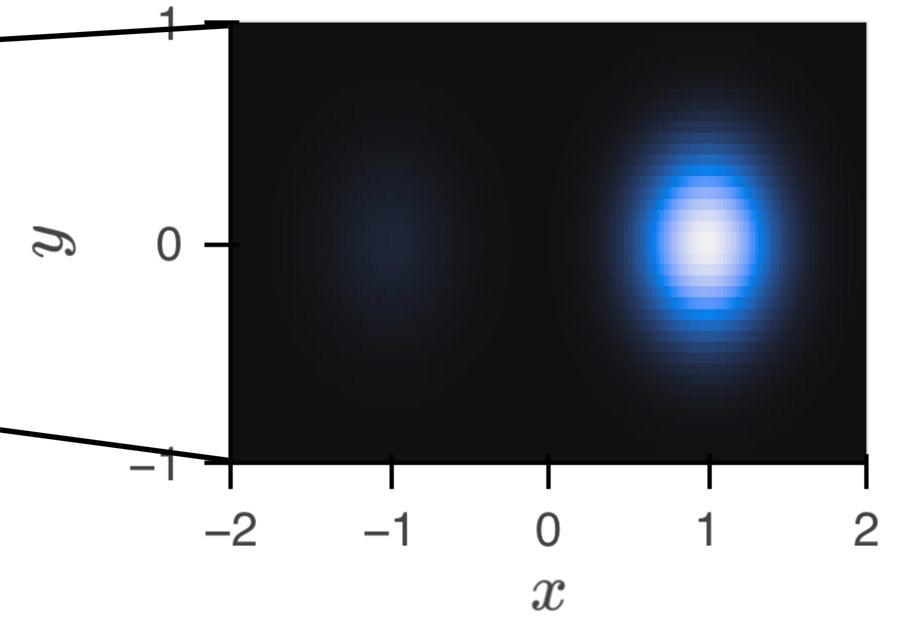
- Phase shift $\Phi = \phi_a + \phi_L$
- Acceleration phase shift

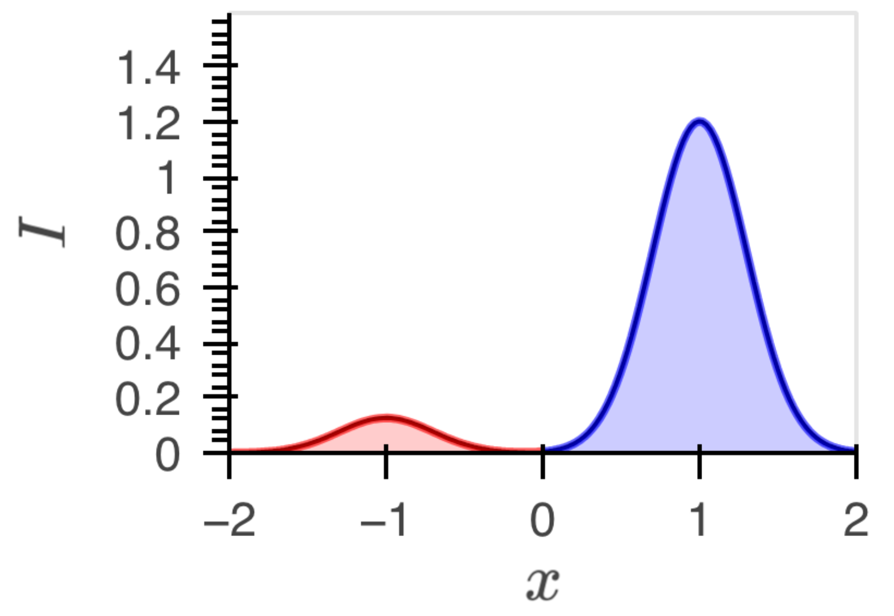
$$\phi_a = kaT^2$$

- Population $\propto \cos(\Phi)$
- Extract population via Gaussian image model

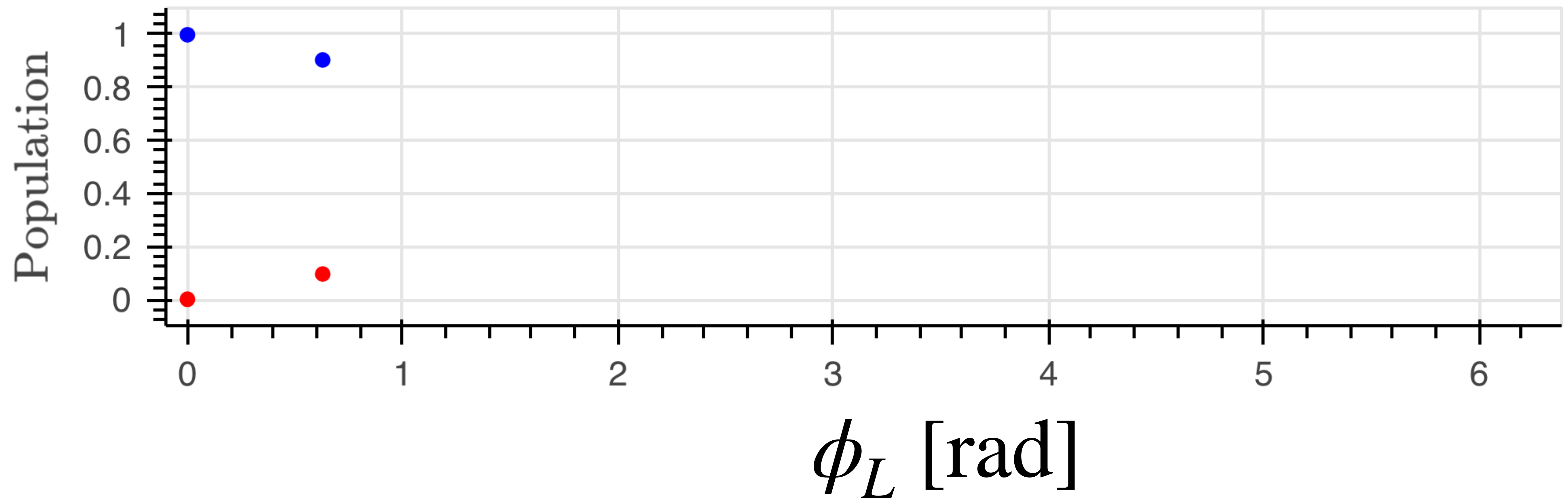
Signal Extraction via Phase Scan







$$\phi_L = 0.6 \text{ rad}$$



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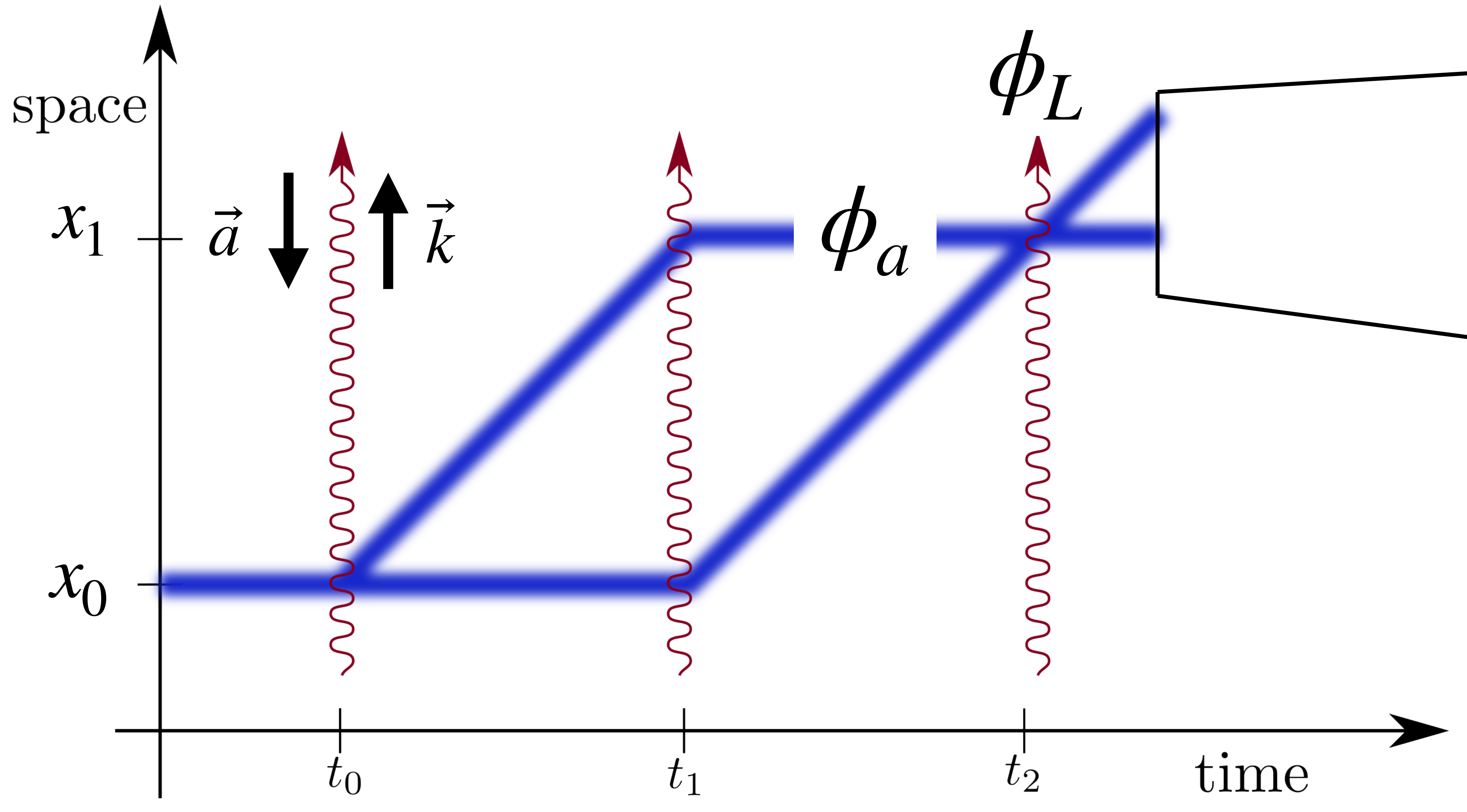
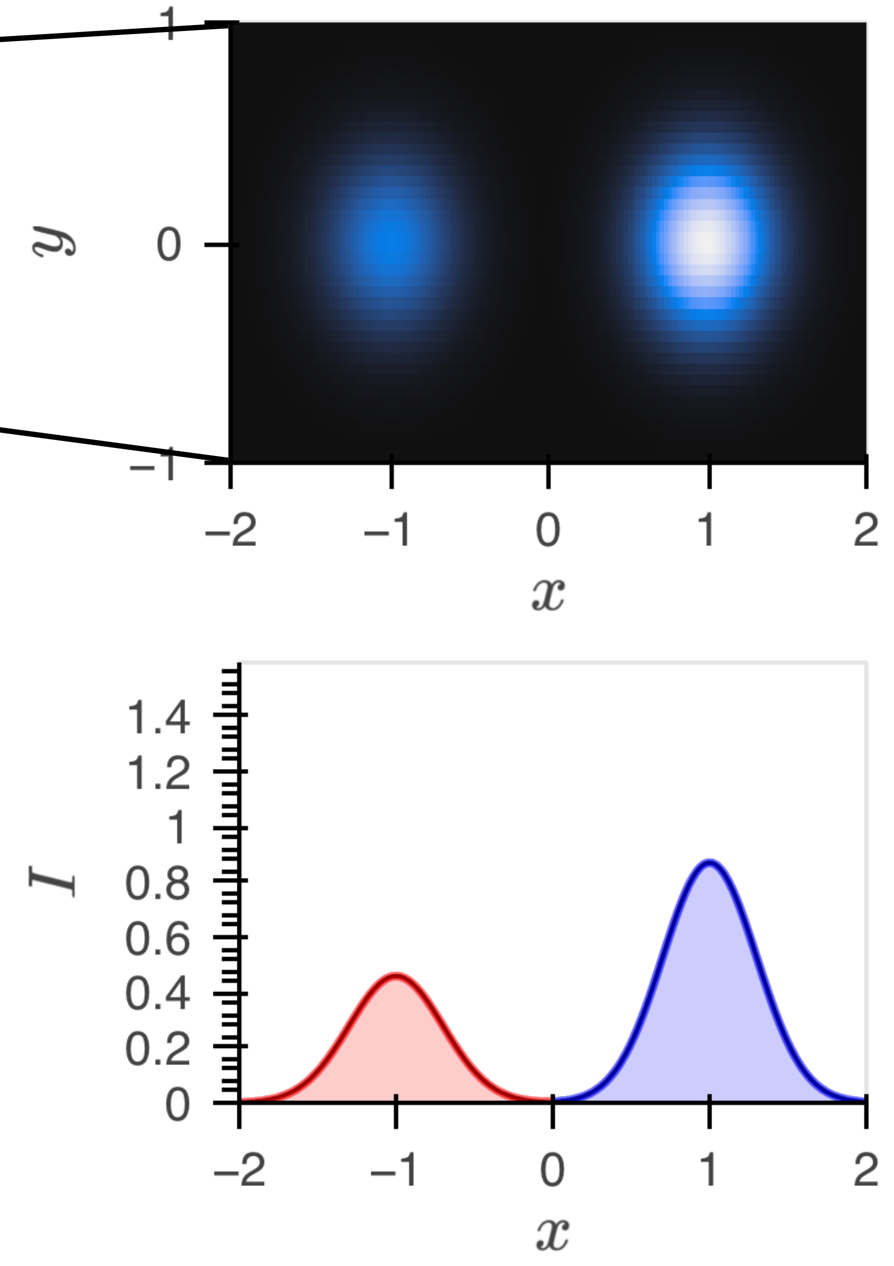
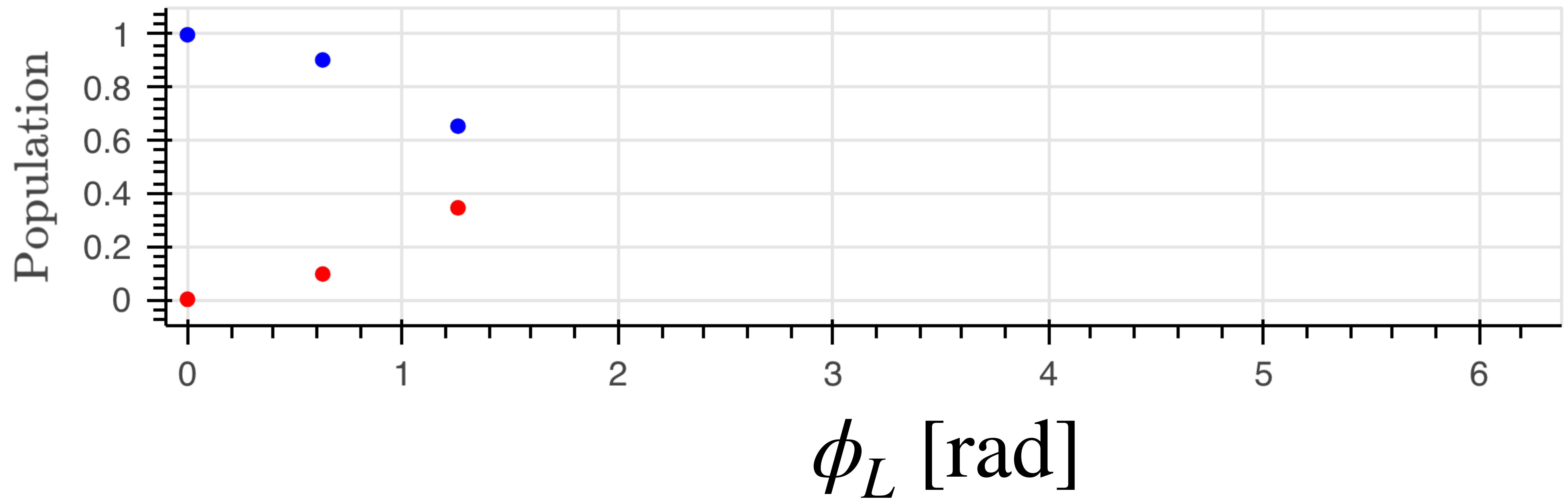
Signal Extraction via Phase Scan





$\phi_L = 1.3$ rad



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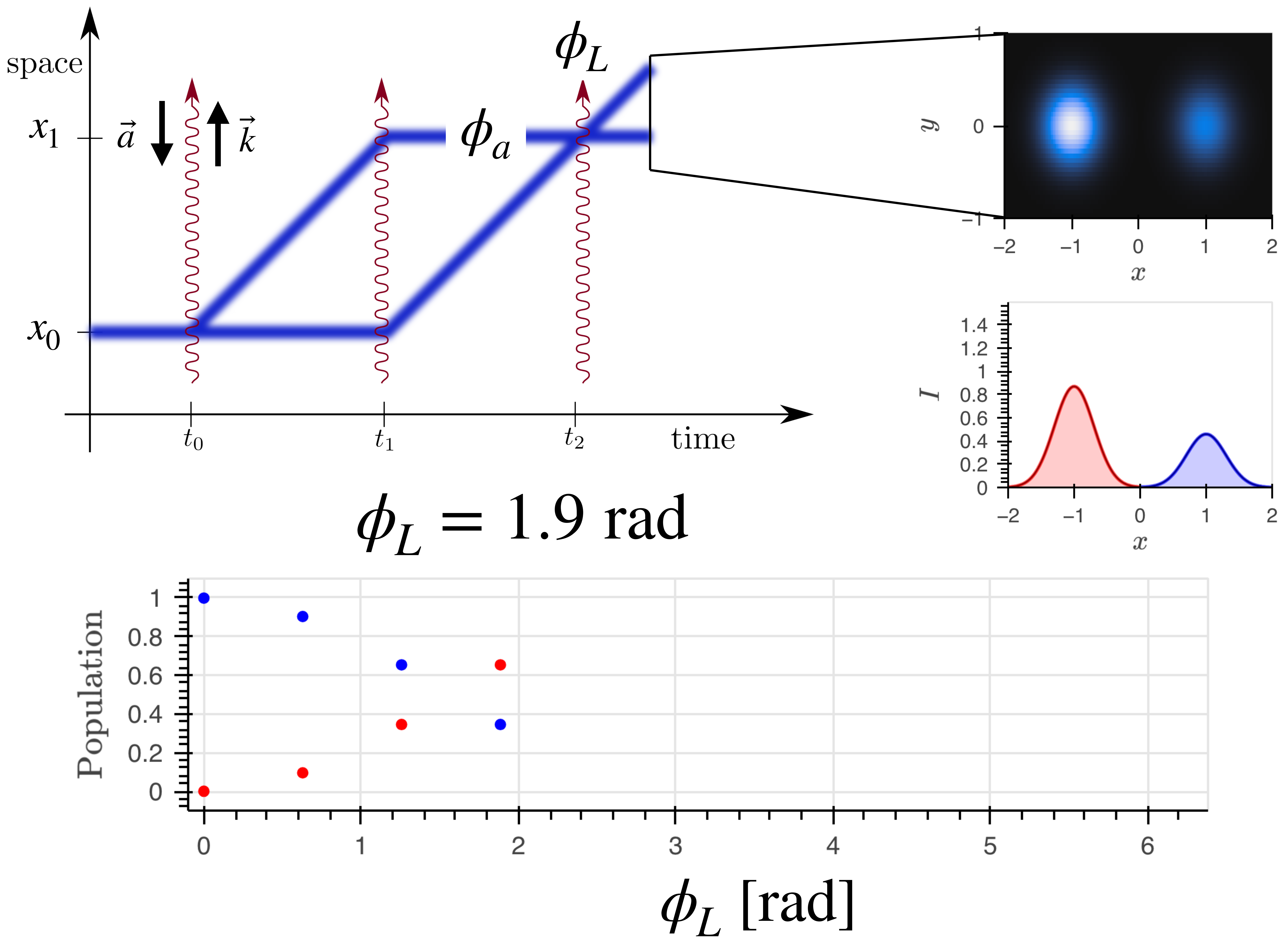
Signal Extraction via Phase Scan



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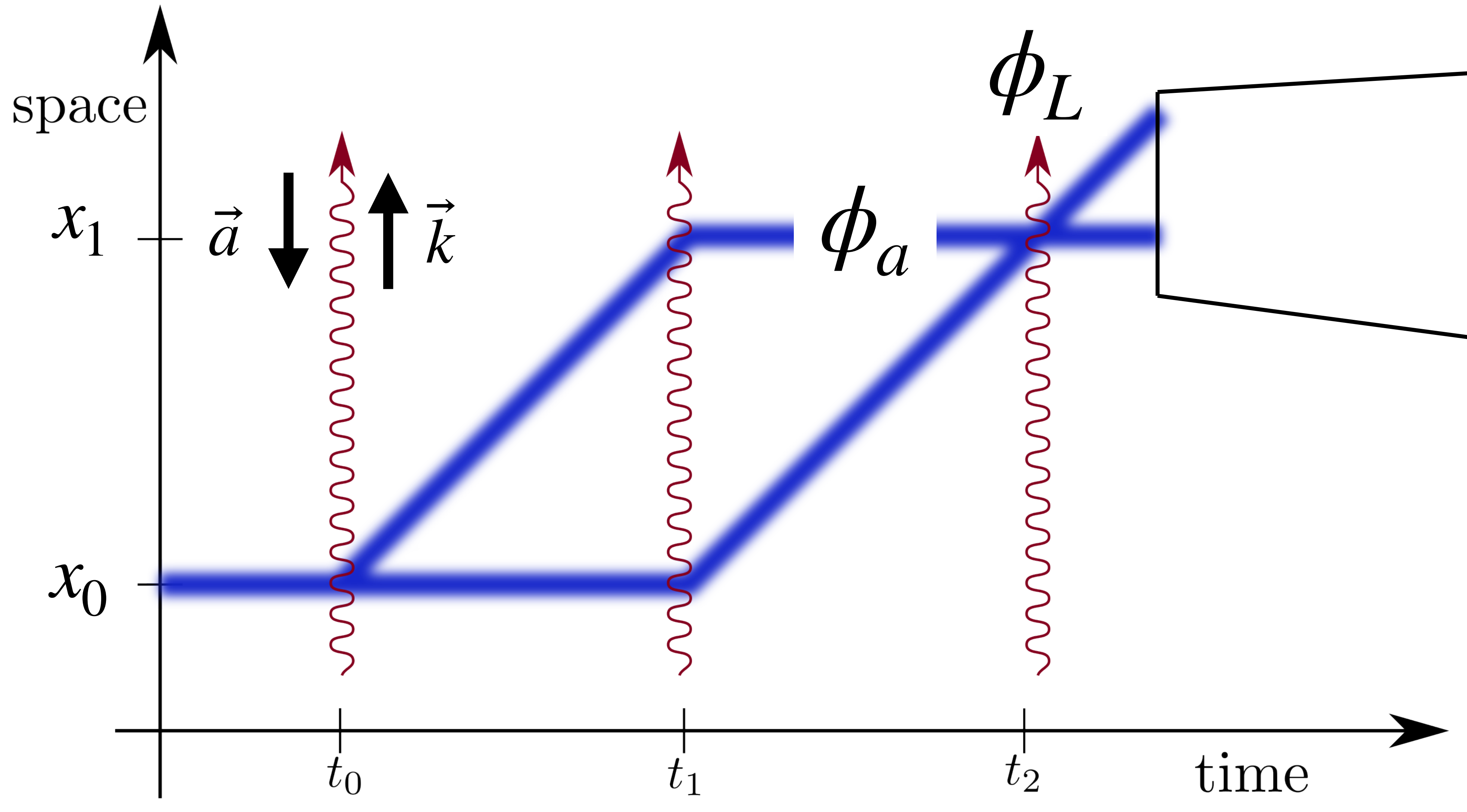
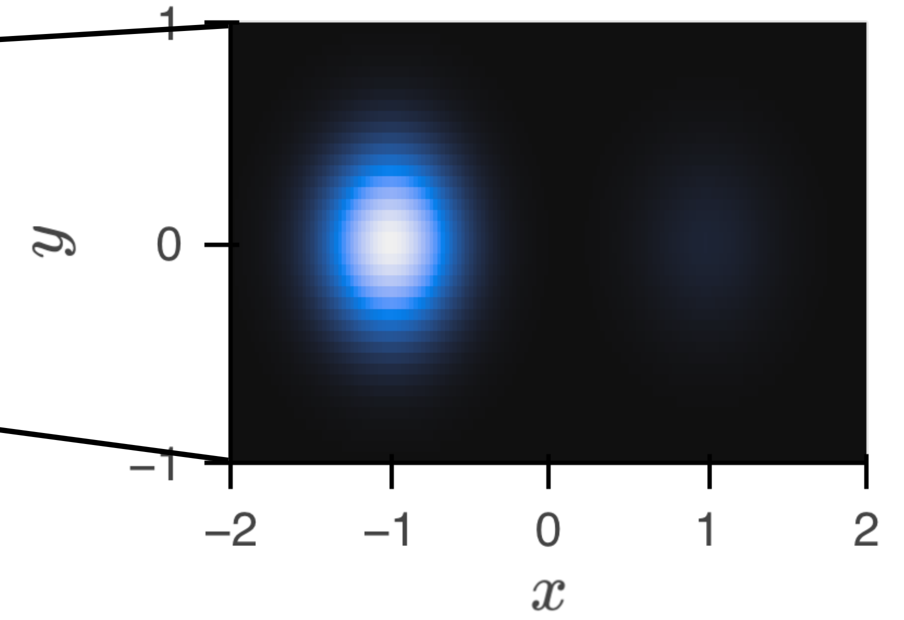
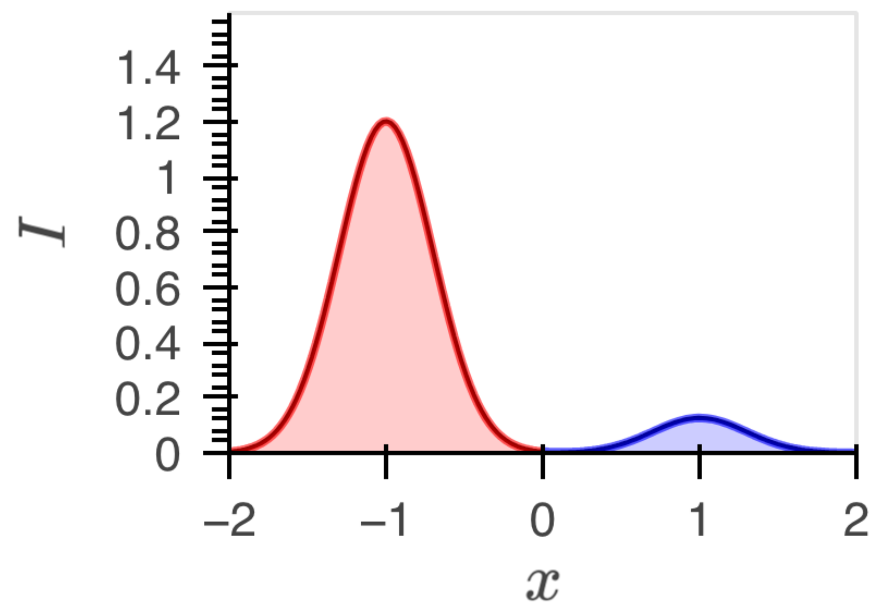
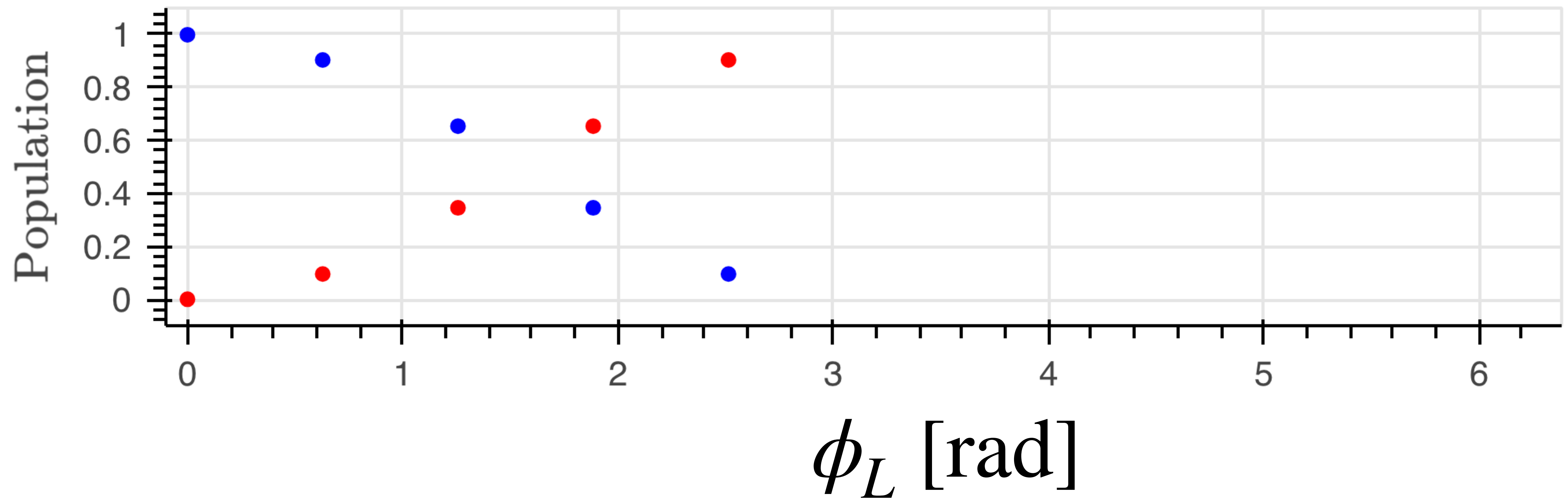






$$\phi_L = 2.5 \text{ rad}$$



- Phase shift $\Phi = \phi_a + \phi_L$
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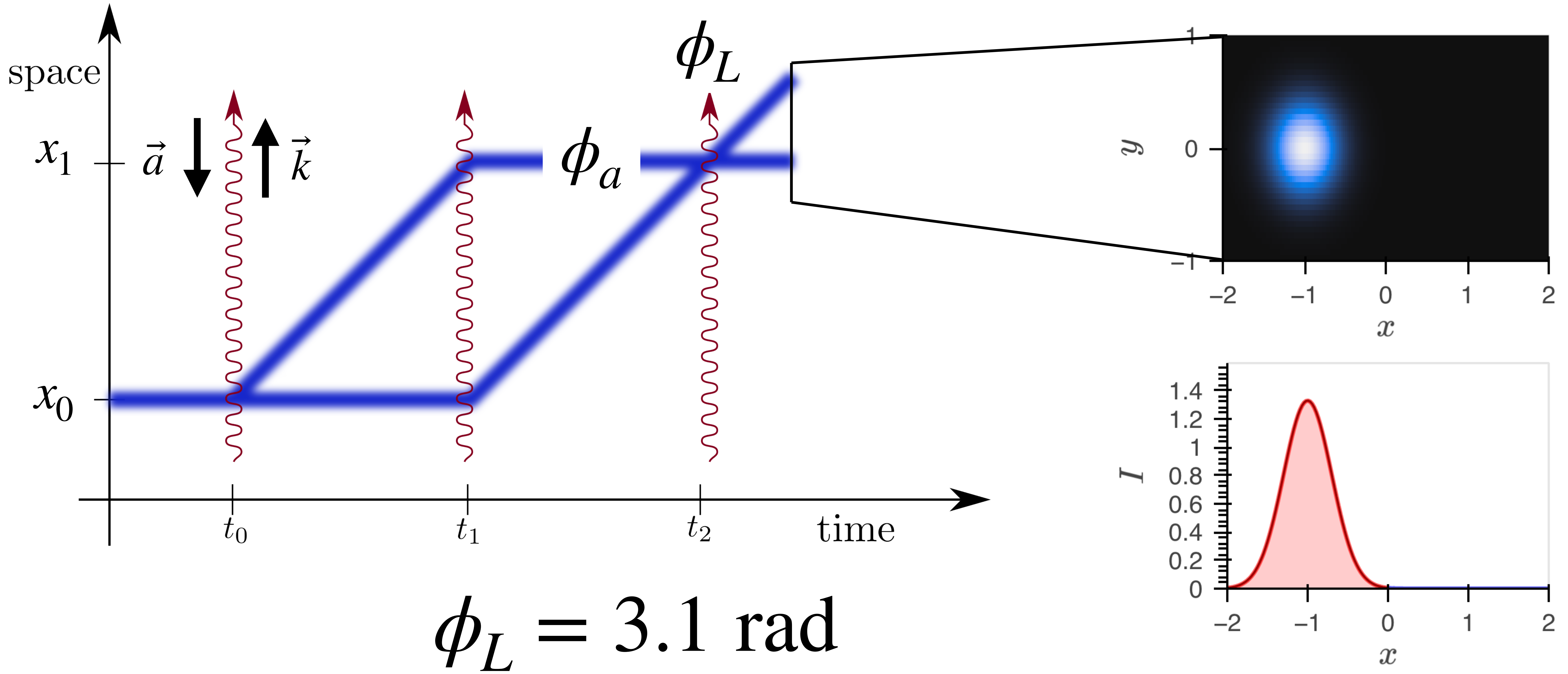
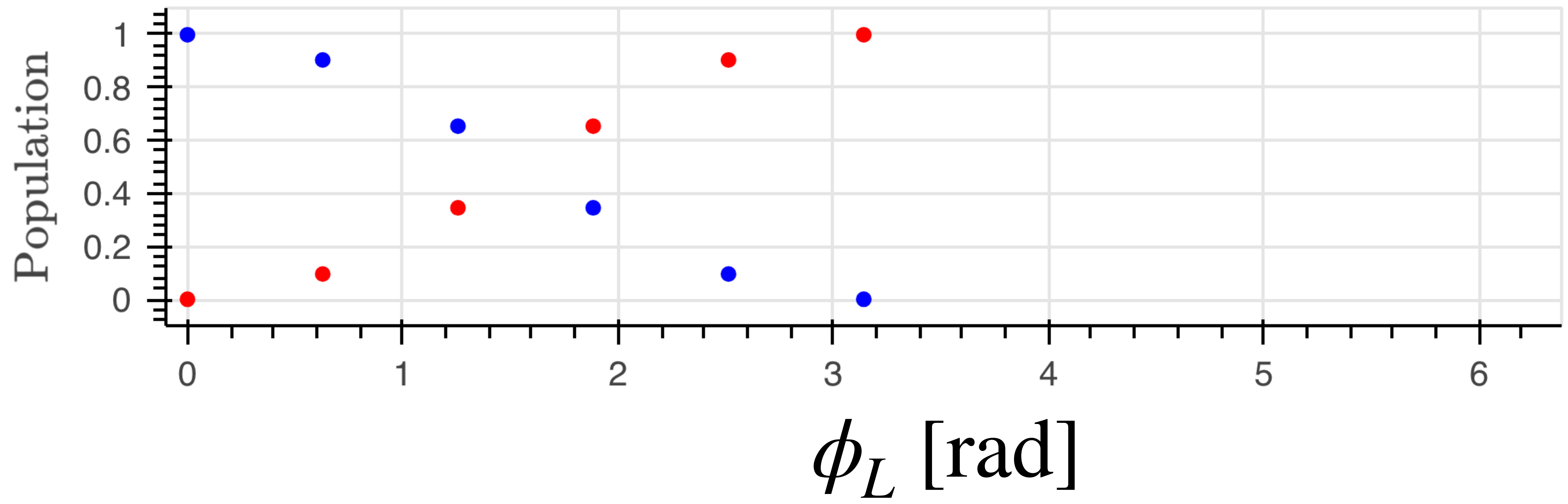


$\phi_L = 3.1$ rad



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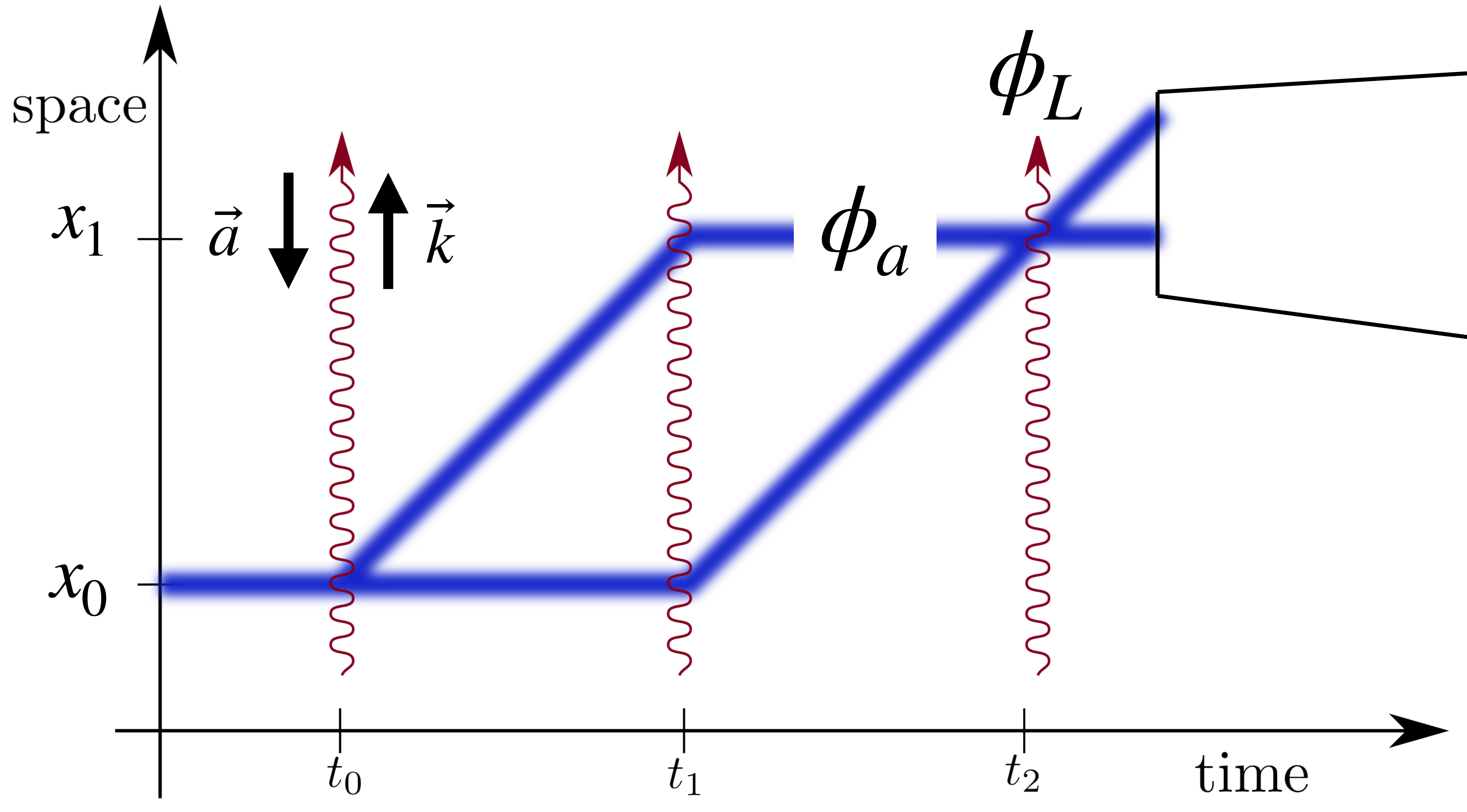
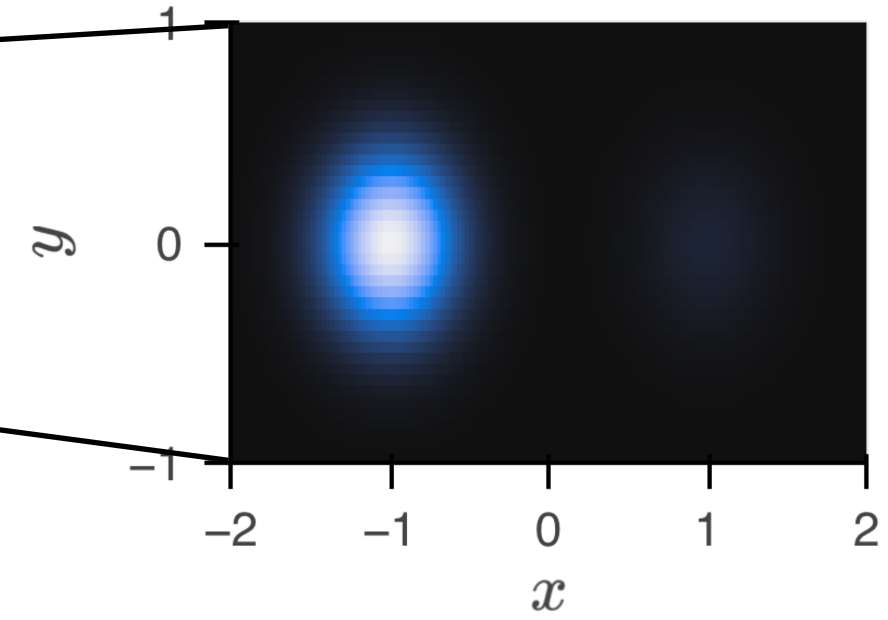
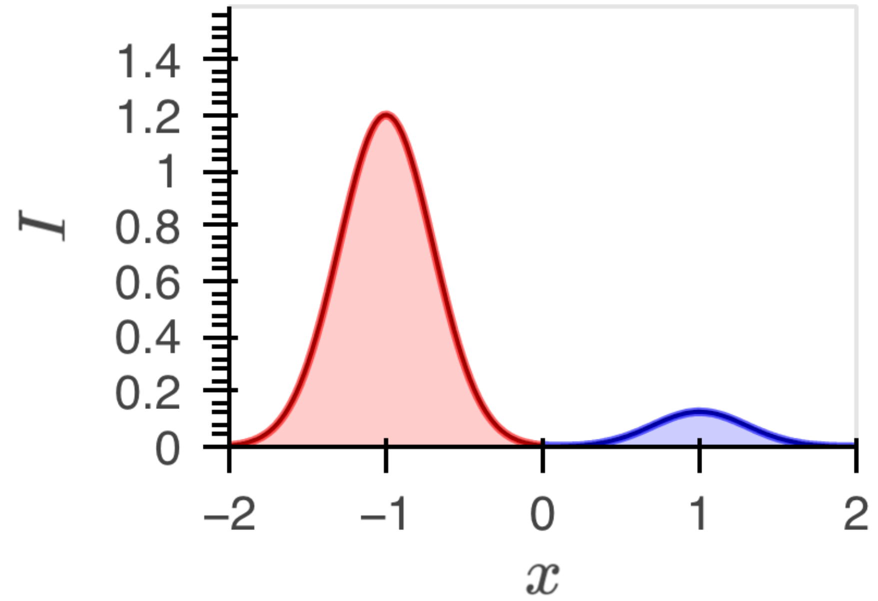
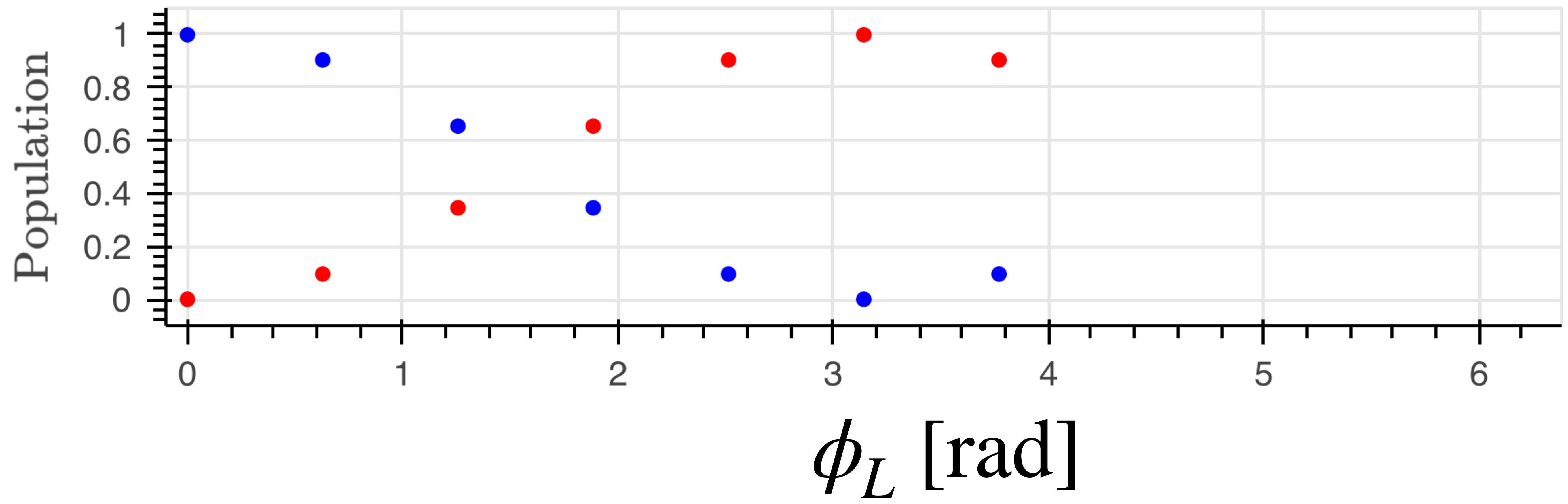






$$\phi_L = 3.8 \text{ rad}$$



- Phase shift $\Phi = \phi_a + \phi_L$
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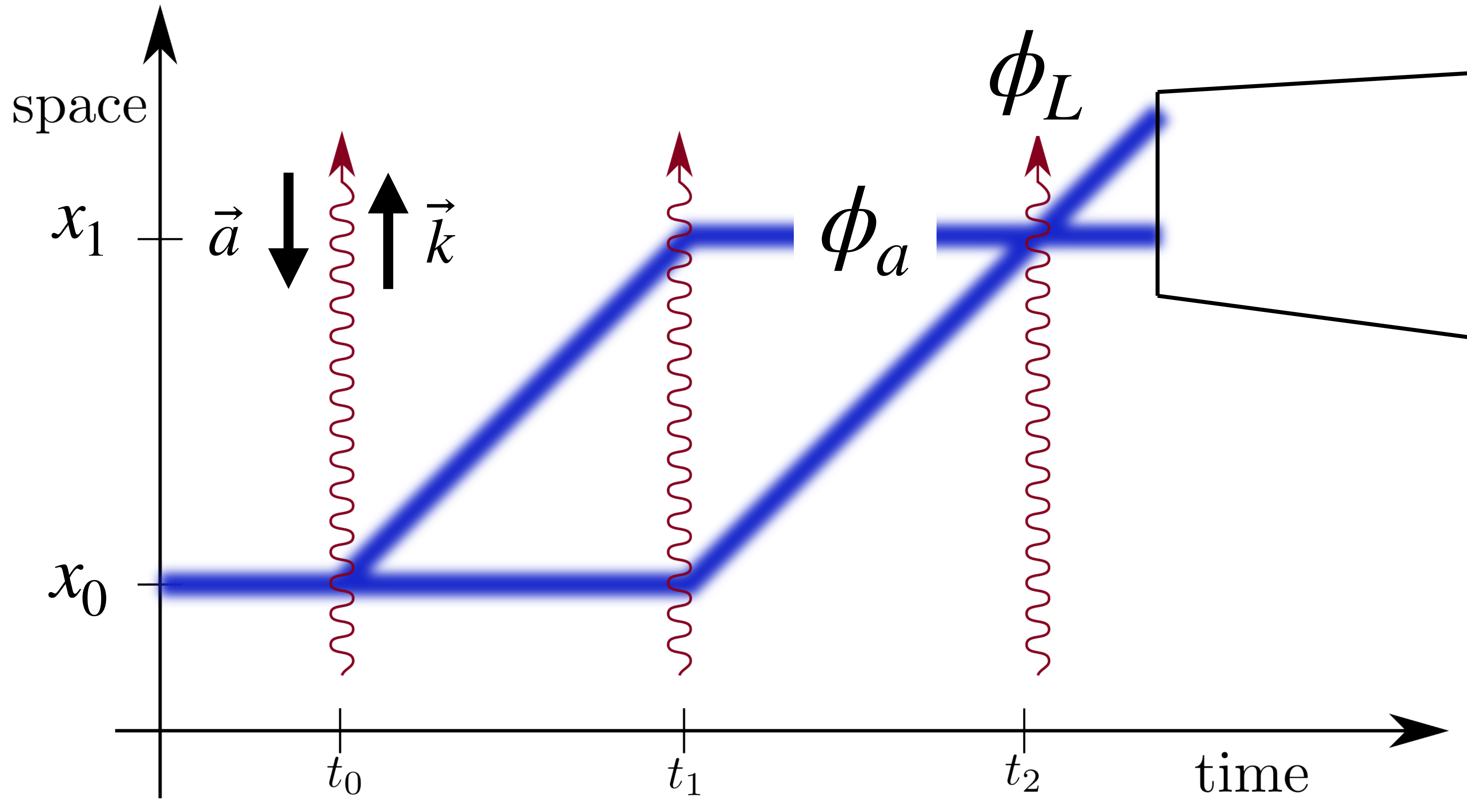
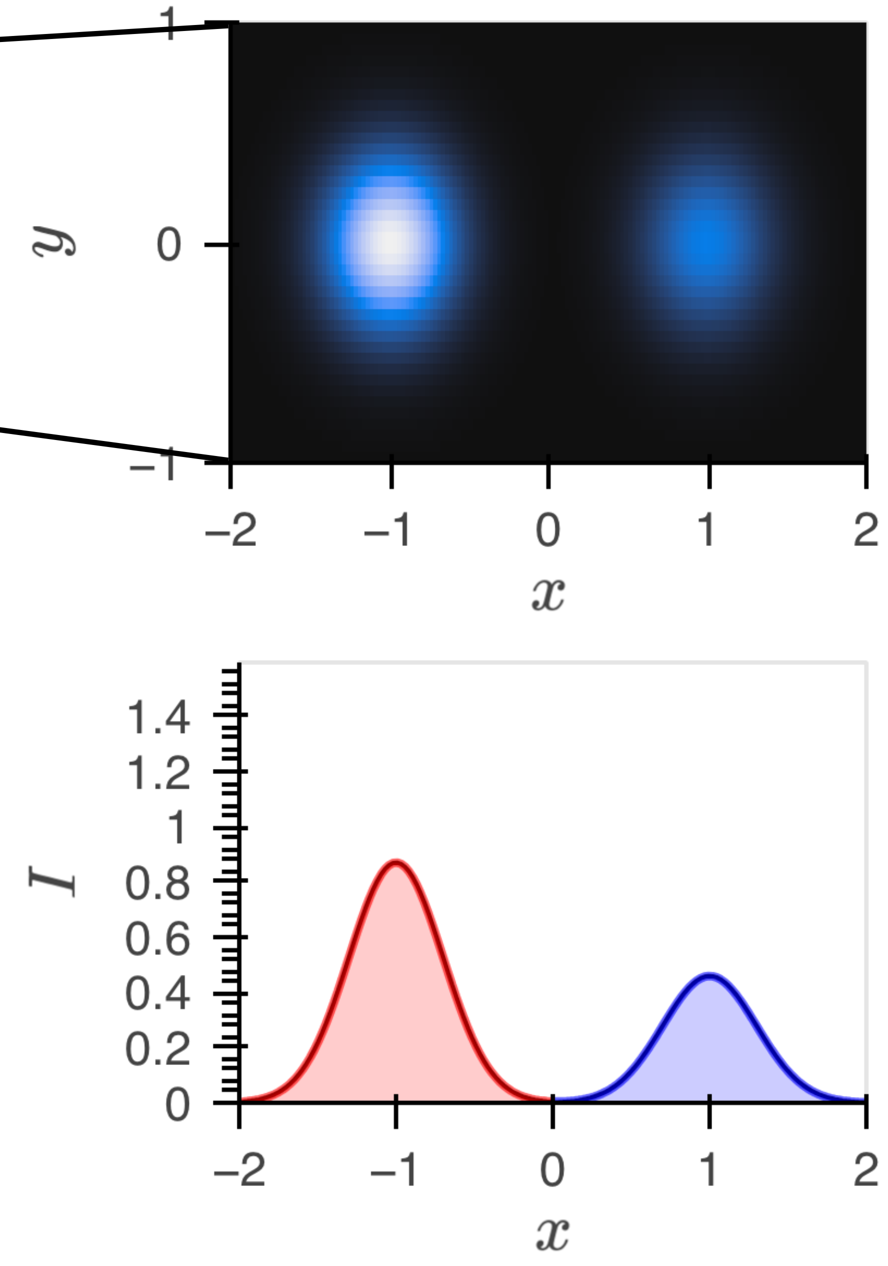
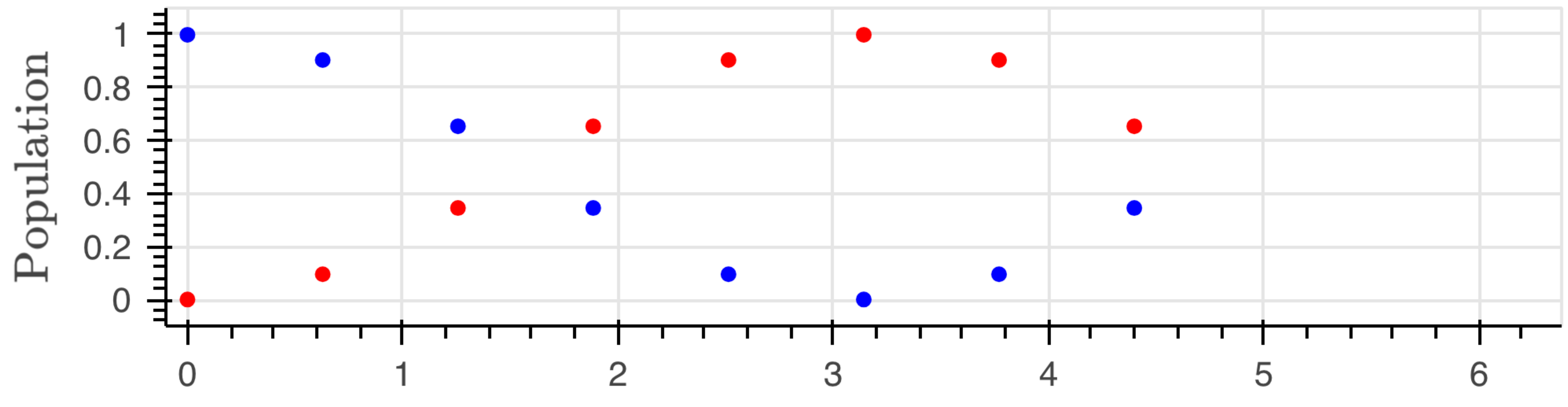
Signal Extraction via Phase Scan





$$\phi_L = 4.4 \text{ rad}$$



- Phase shift $\Phi = \phi_a + \phi_L$
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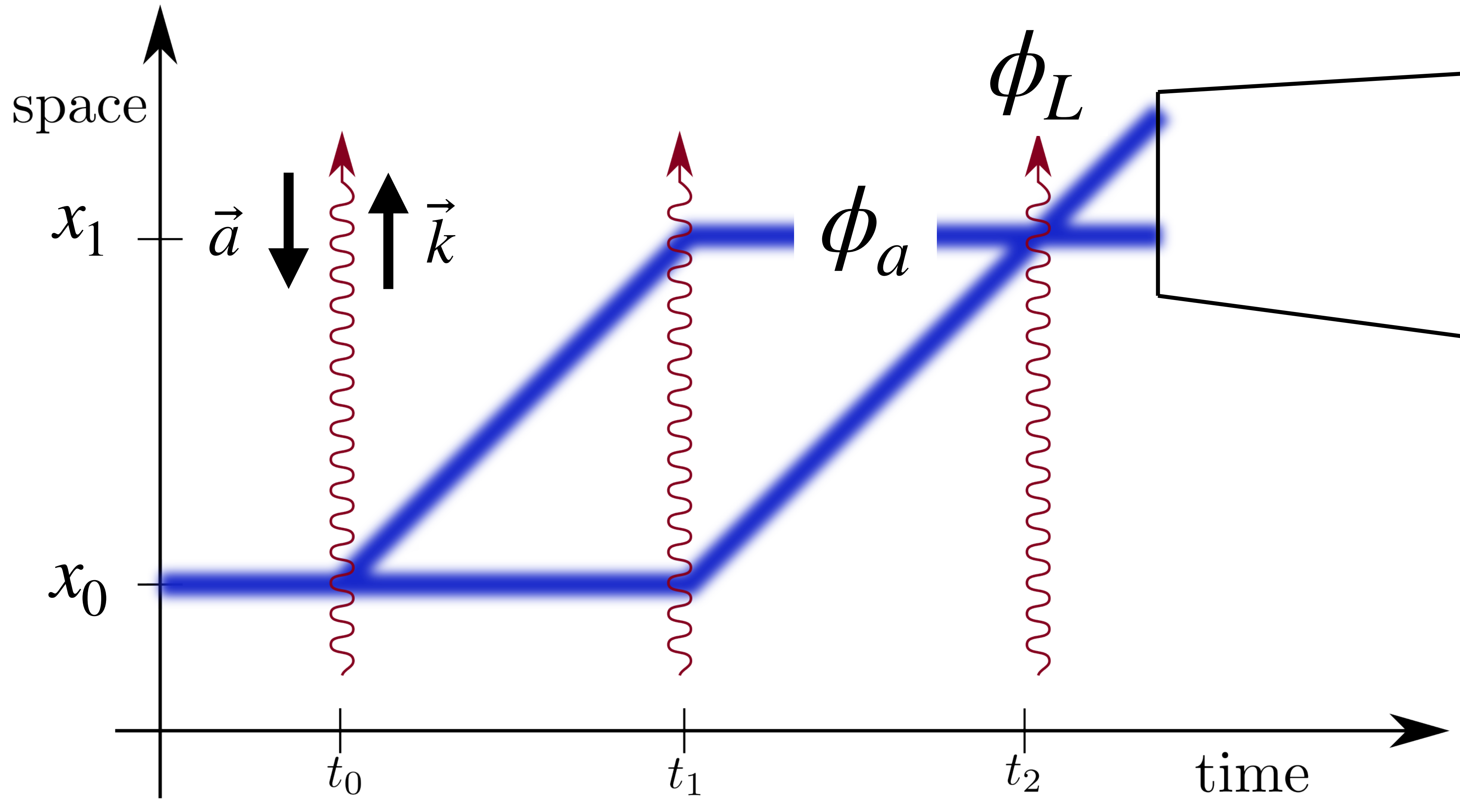
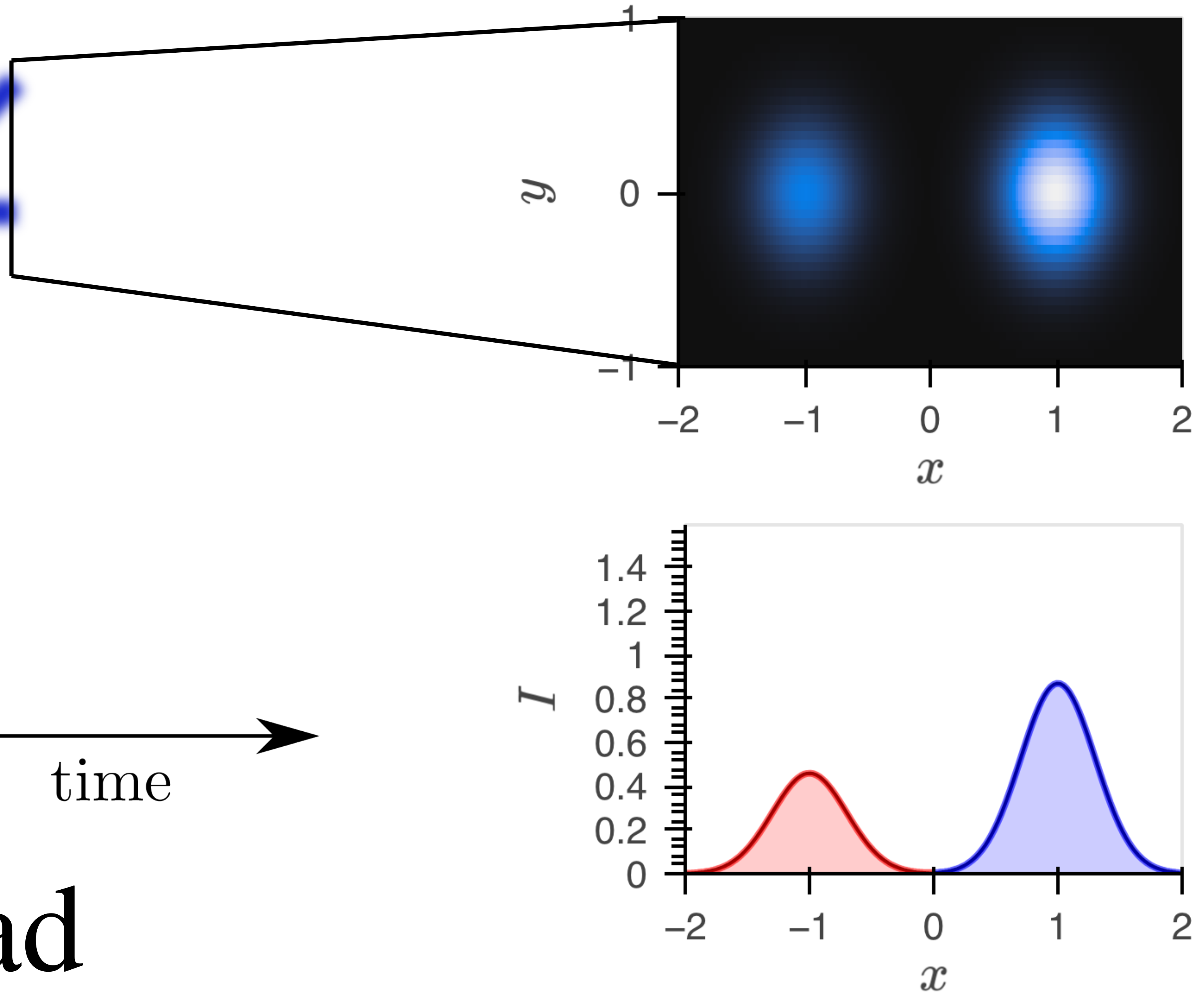
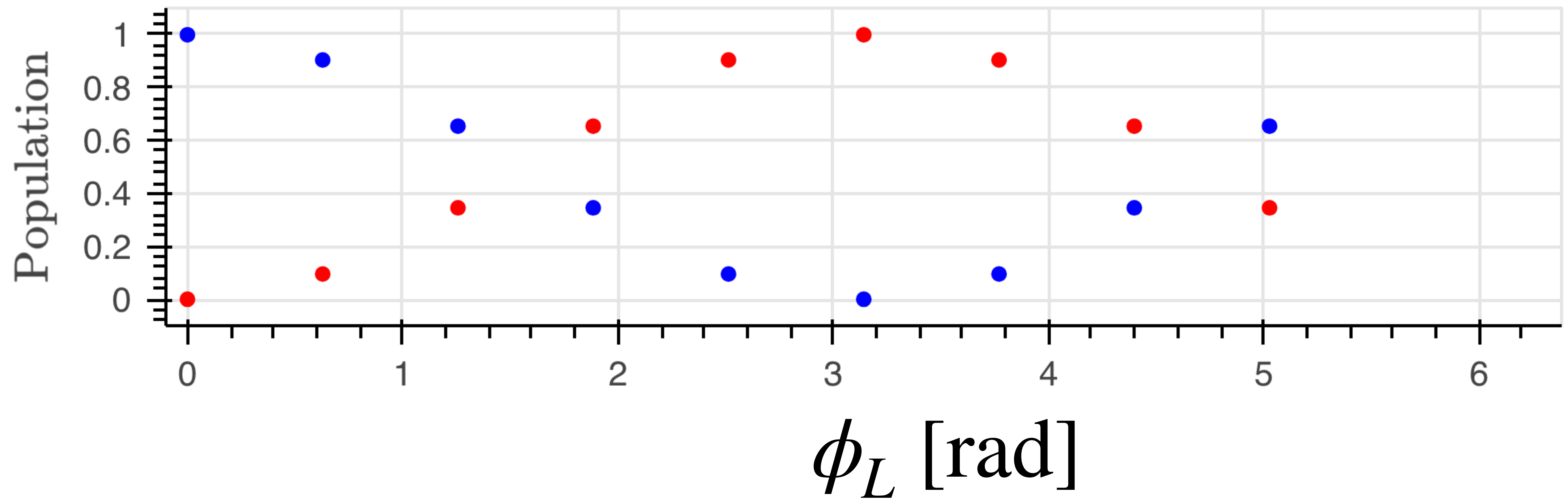
Signal Extraction via Phase Scan





$$\phi_L = 5.0 \text{ rad}$$

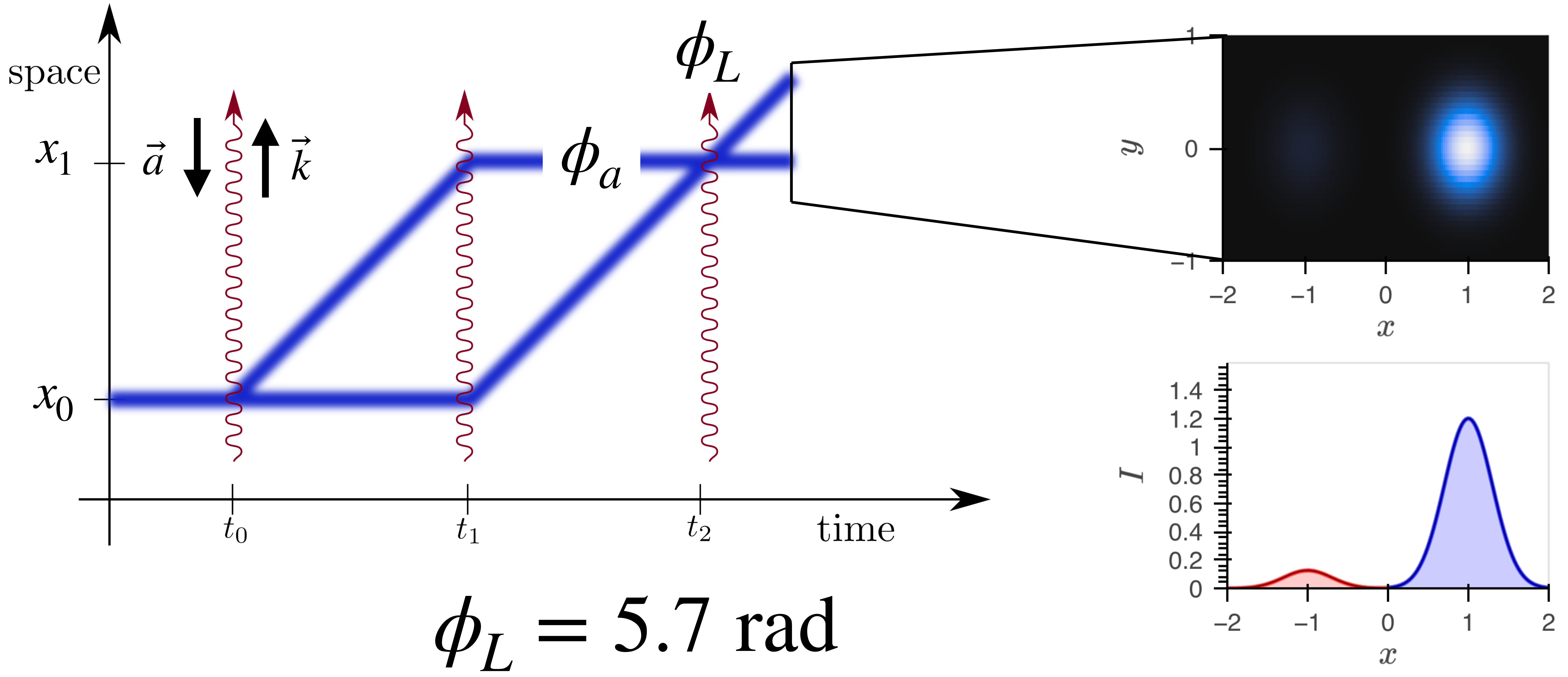


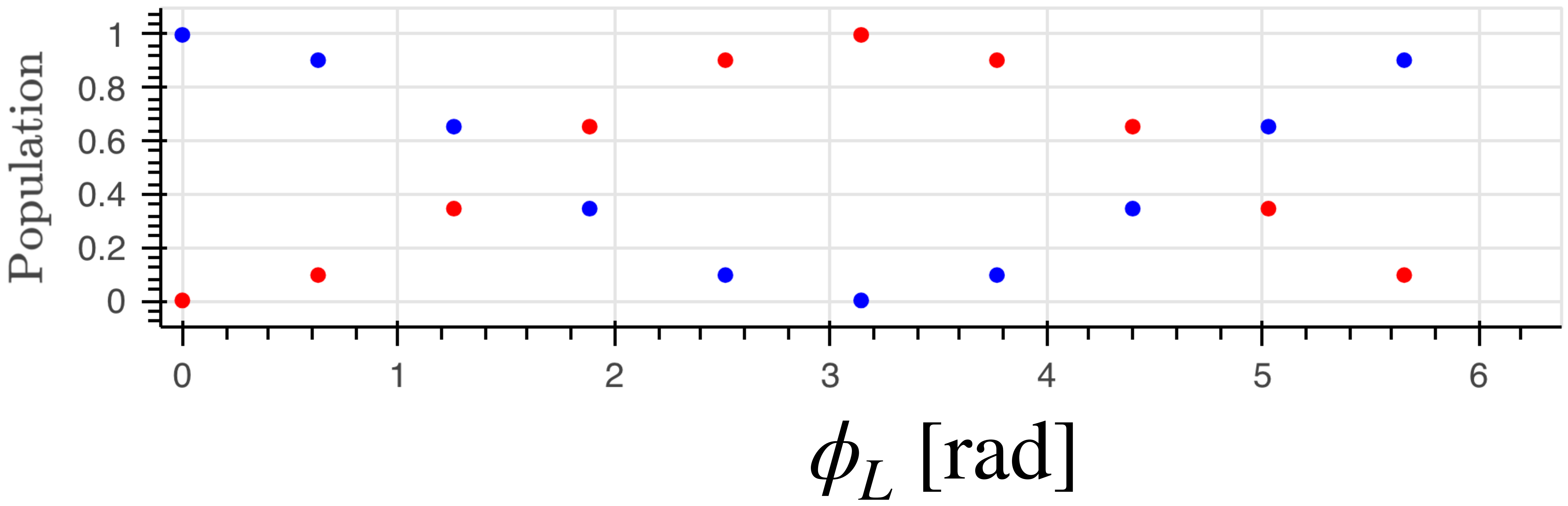
- Phase shift $\Phi = \phi_a + \phi_L$
- Acceleration phase shift

$$\phi_a = kaT^2$$

- Population $\propto \cos(\Phi)$
- Extract population via Gaussian image model

Signal Extraction via Phase Scan





- Phase shift $\Phi = \phi_a + \phi_L$
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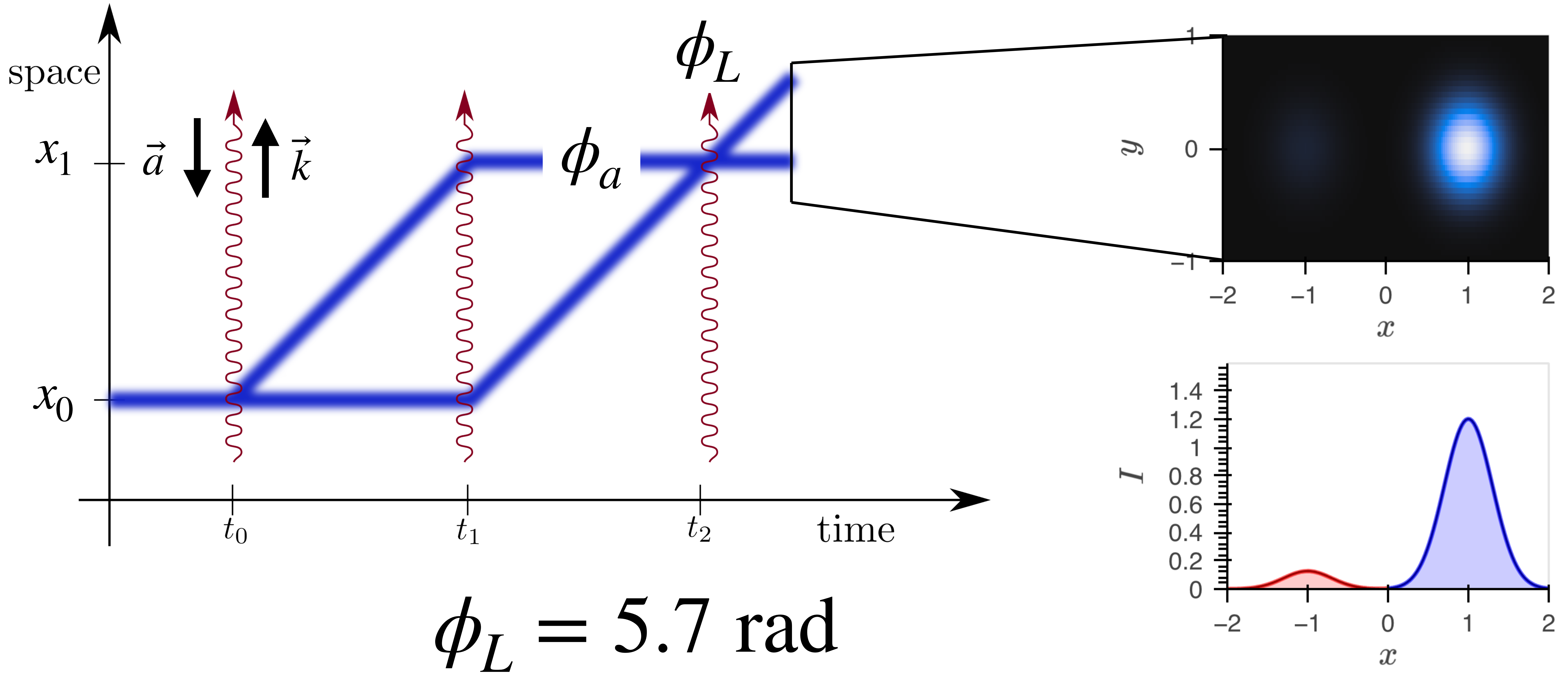
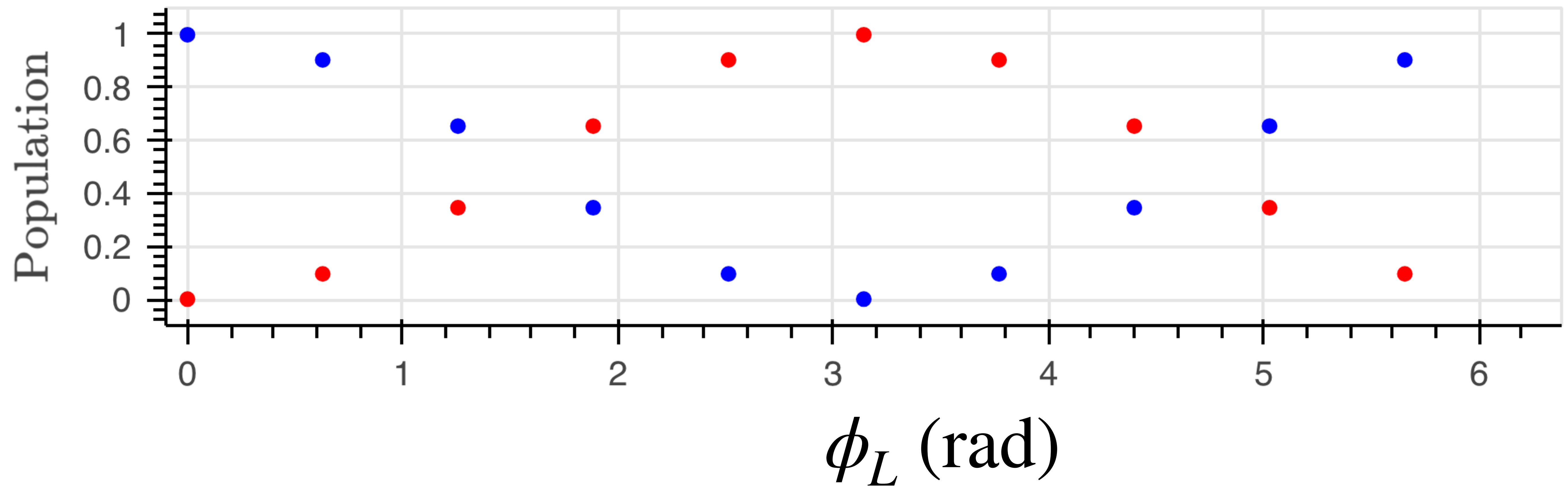
Signal Extraction via Phase Scan



$$\phi_L = 5.7 \text{ rad}$$



- Phase shift $\Phi = \phi_a + \phi_L$
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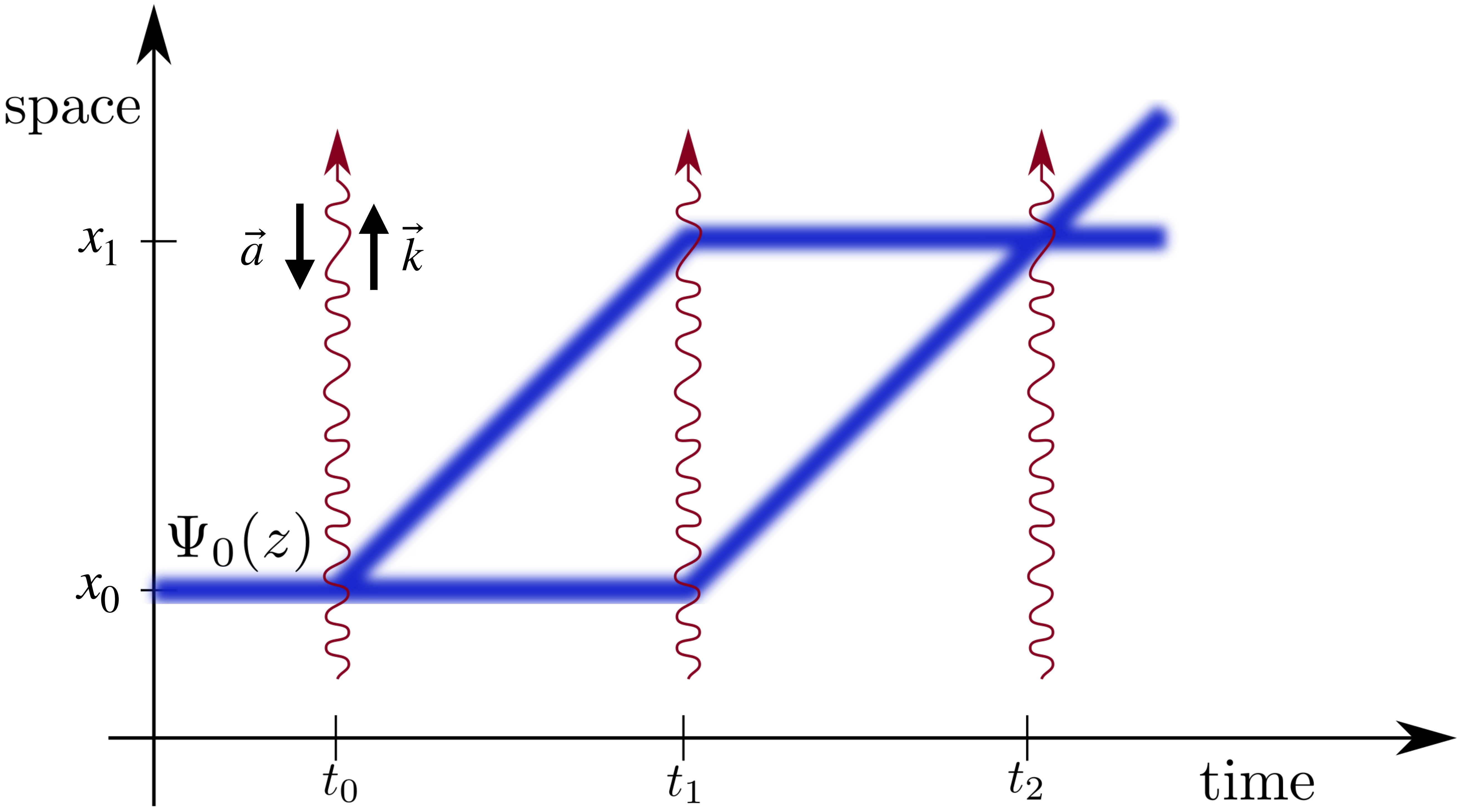
Wavefront Aberrations

- Can imprint spatially dependent phase profiles



Wavefront Aberrations

- Can imprint spatially dependent phase profiles
- Often the leading systematic uncertainty

Wave-front curvature	0.6	0.3
Wave-front distortion	3.9	1.9
Gouy phase	108.2	5.4

- Their impact depends on the data analysis method

Table 1 | Error budget on *α*

Source	Correction (×10^{-11})	Relative uncertainty (×10^{-11})
Gravity gradient	−0.6	0.1
Alignment of the beams	0.5	0.5
Coriolis acceleration		1.2
Frequencies of the lasers		0.3
Wave-front curvature	0.6	0.3
Wave-front distortion	3.9	1.9
Gouy phase	108.2	5.4
Residual Raman light shift	2.3	2.3
Index of refraction	0	<0.1
Internal interaction	0	<0.1
Light shift (two-photon transition)	−11.0	2.3
Second-order Zeeman effect		0.1
Phase shifts in Raman phase-lock loop	−39.8	0.6
Global systematic effects	**64.2**	**6.8**
Statistical uncertainty		**2.4**
Relative mass of ^{87}Rb[a]: 86.9091805310(60)		3.5
Relative mass of the electron[b]: 5.48579909065(16) × 10^{-4}		1.5
Rydberg constant[b]: 10,973,731.568160(21) m^{-1}		0.1
Total: α^{-1} = 137.035999206(11)		**8.1**

For each systematic effect, more discussion can be found in Methods.
[a]From ref. [13].
[b]From https://pml.nist.gov/cuu/Constants/.

Morel, L., Yao, Z., Cladé, P. *et al.* Determination of the fine-structure constant with an accuracy of 81 parts per trillion. *Nature* **588**, 61–65 (2020). https://doi.org/10.1038/s41586-020-2964-7

Signal Extraction with Aberrations

- Intensity model

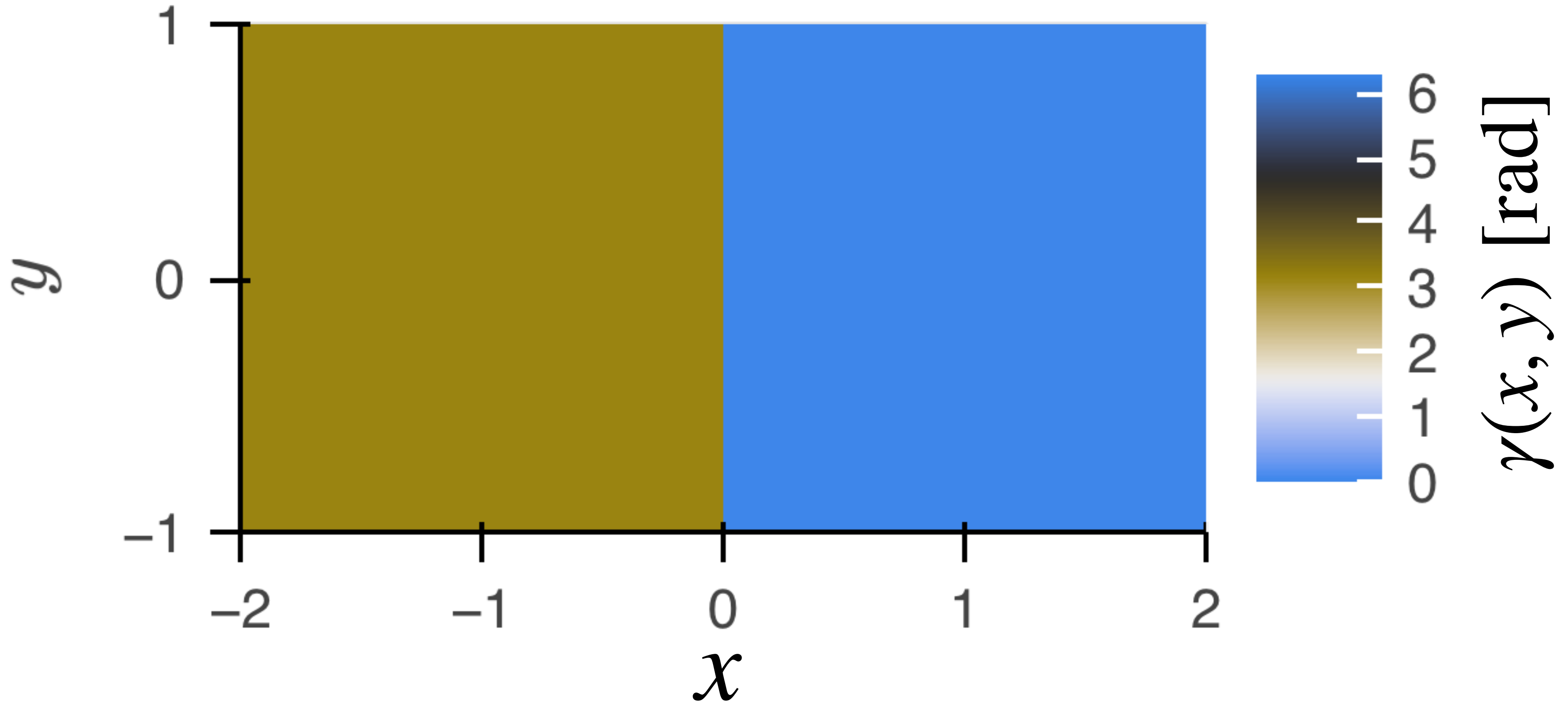
$$I(i,x,y) = \frac{1}{2}\mathrm{env(x,y)}(1 + C\cos[\theta(i) + \gamma(x,y)])$$

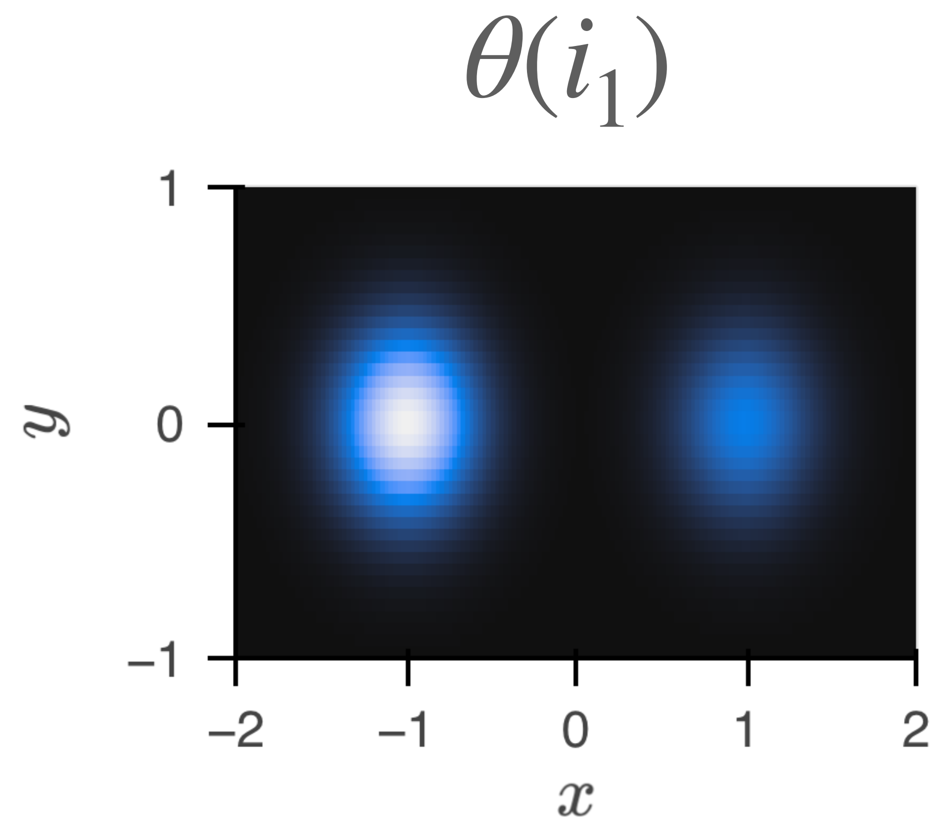
- Per image phase

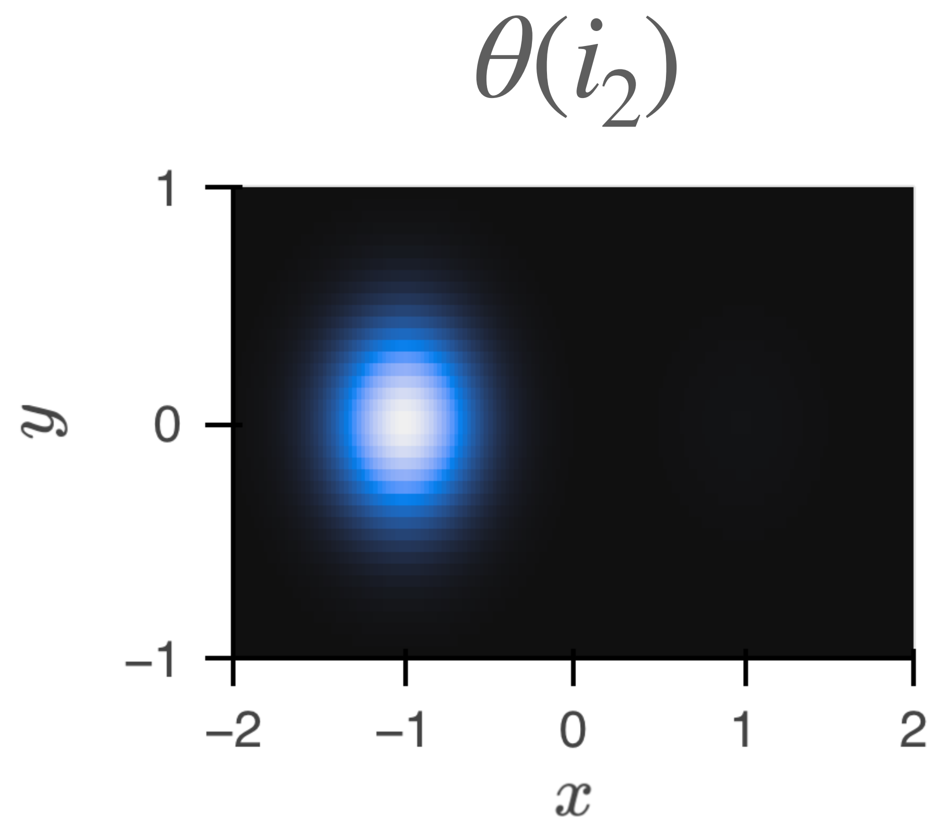
$$\theta(i) = \frac{2\pi i}{50} \text{ with } i \in [0,50) \quad \rightarrow f(\phi_a, \phi_L)$$

- Spatial phase

$$\gamma(x,y) = \begin{cases} 0 & \text{if } x \geq 0 \\ \pi & \text{if } x < 0 \end{cases}$$







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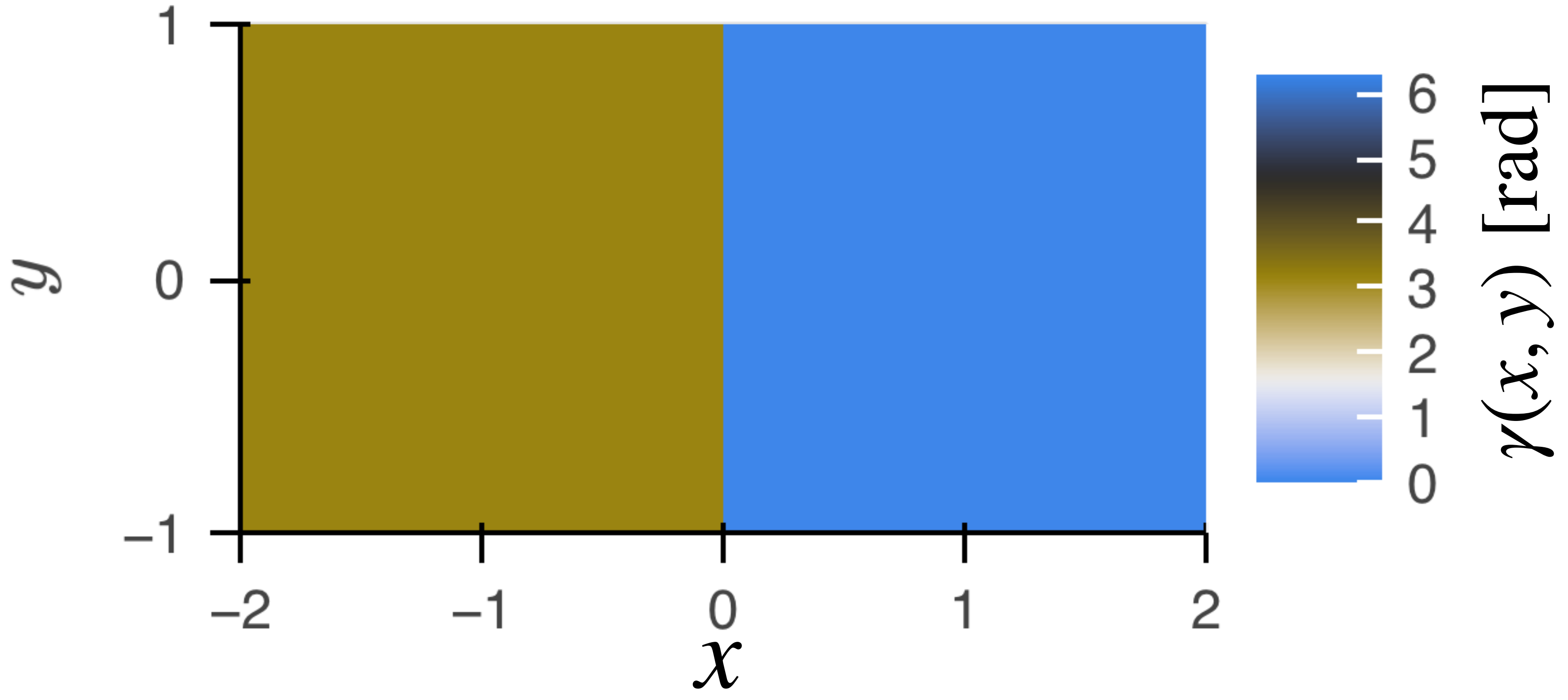
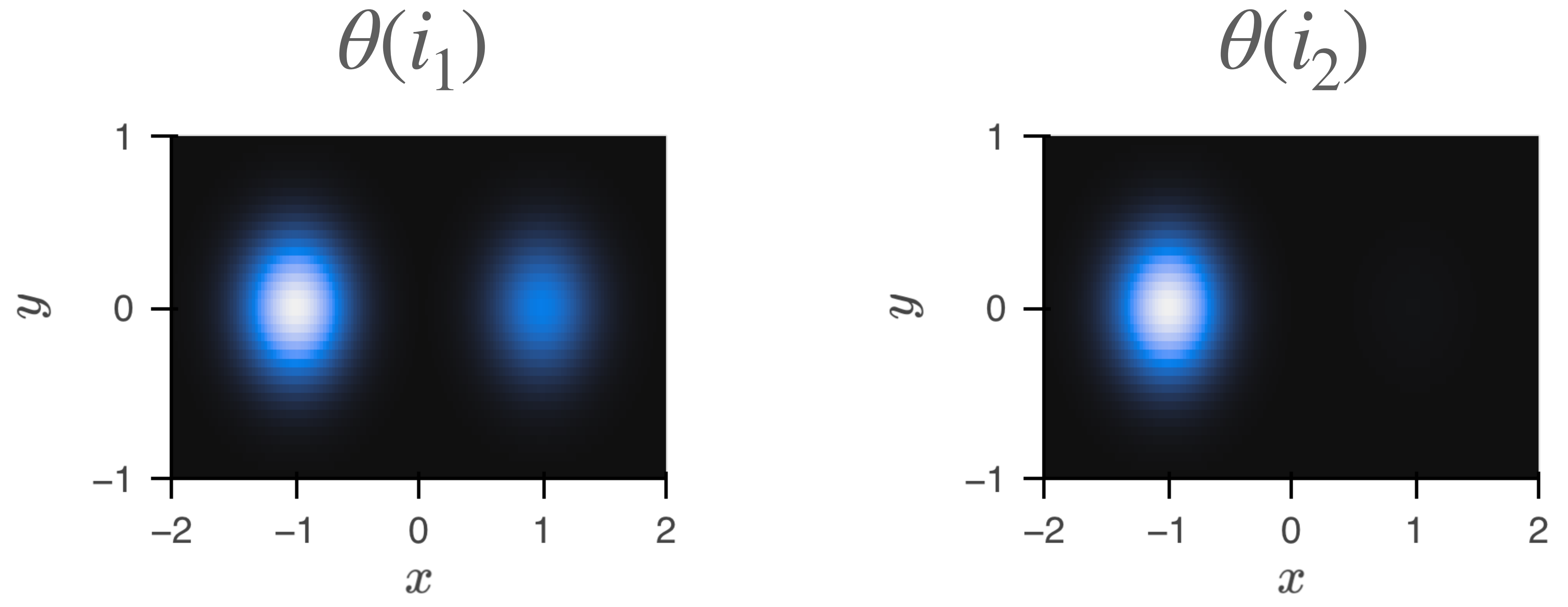
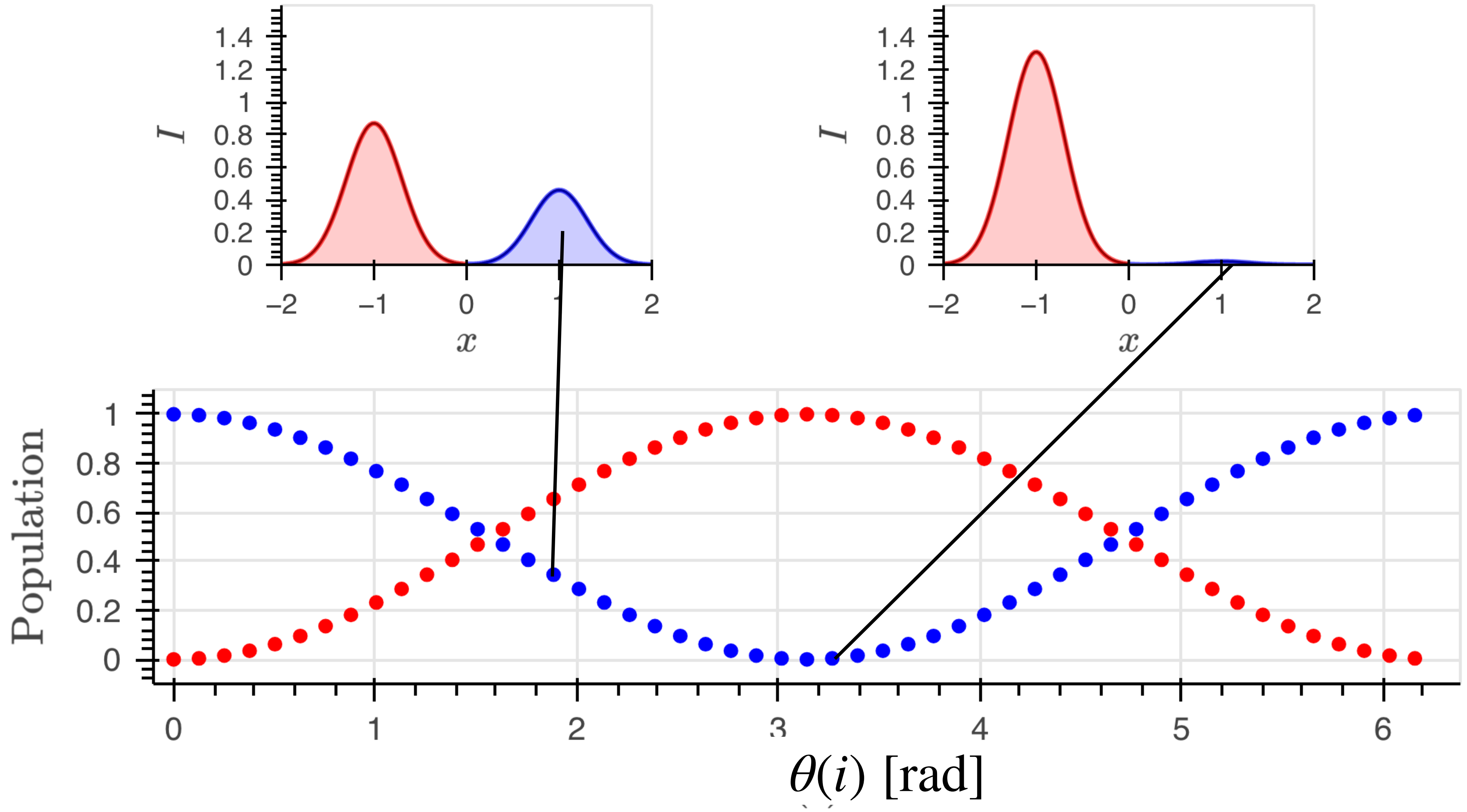




- Signal extraction via Gaussian integrals



Signal Extraction with Aberrations

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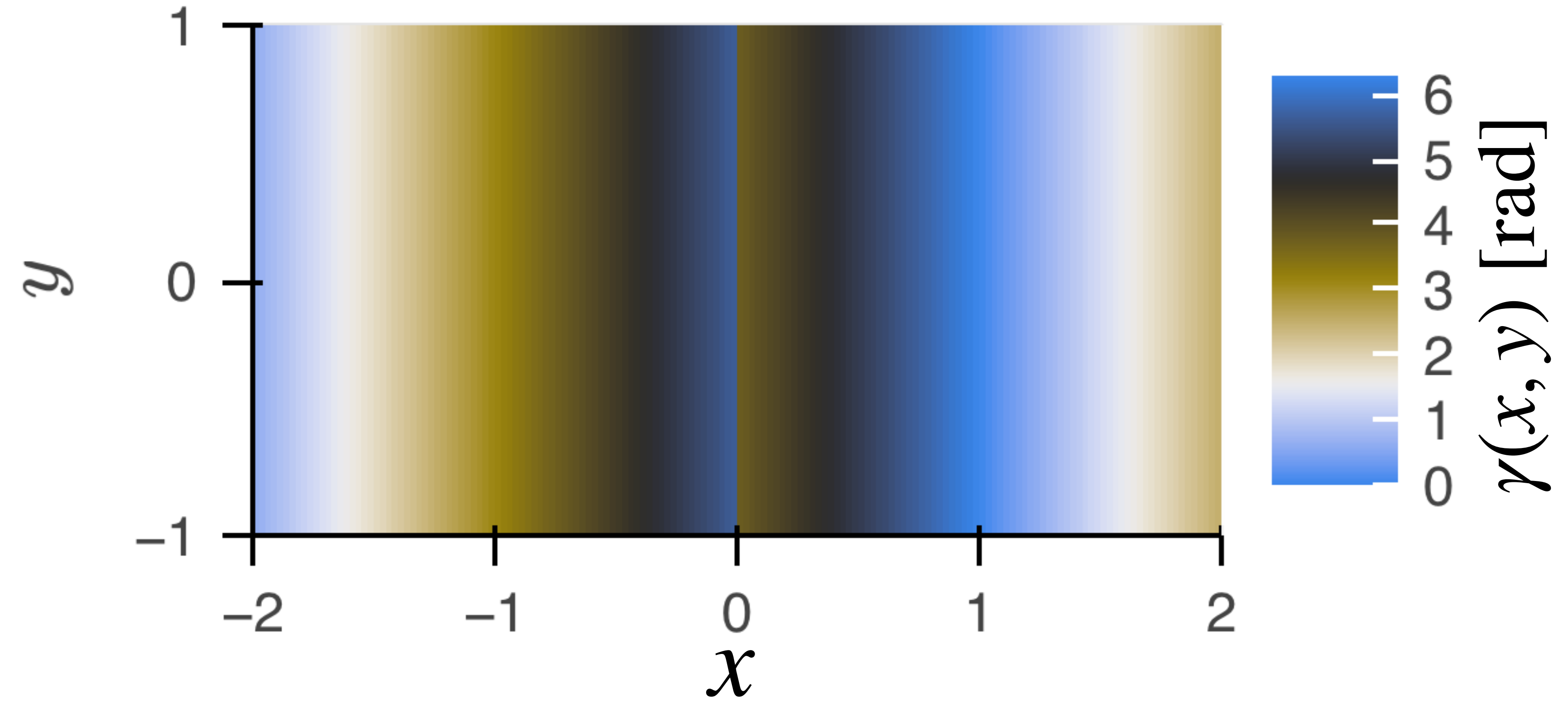
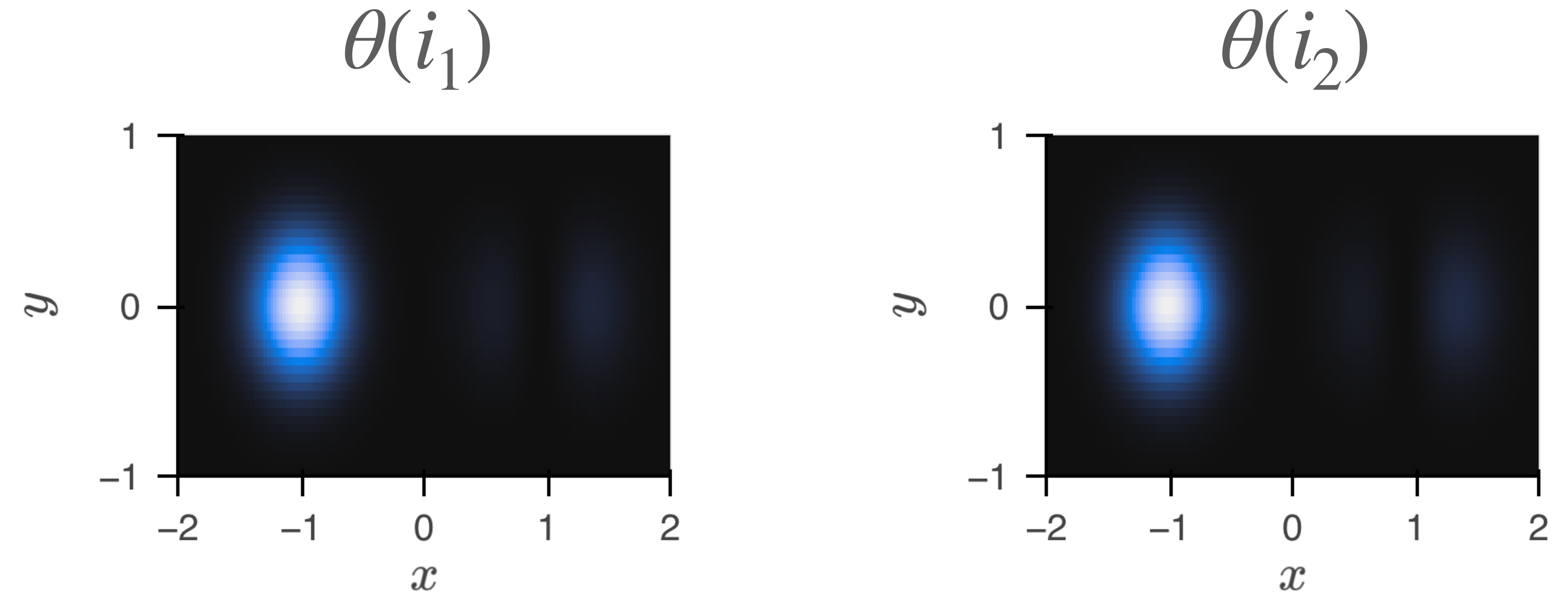
- Spatial phase

$$\gamma(x,y) = \begin{cases} 2.5(x - \delta x) & \text{if } x \geq 0 \\ 2.5(x + \delta x) + \pi & \text{if } x < 0 \end{cases}$$





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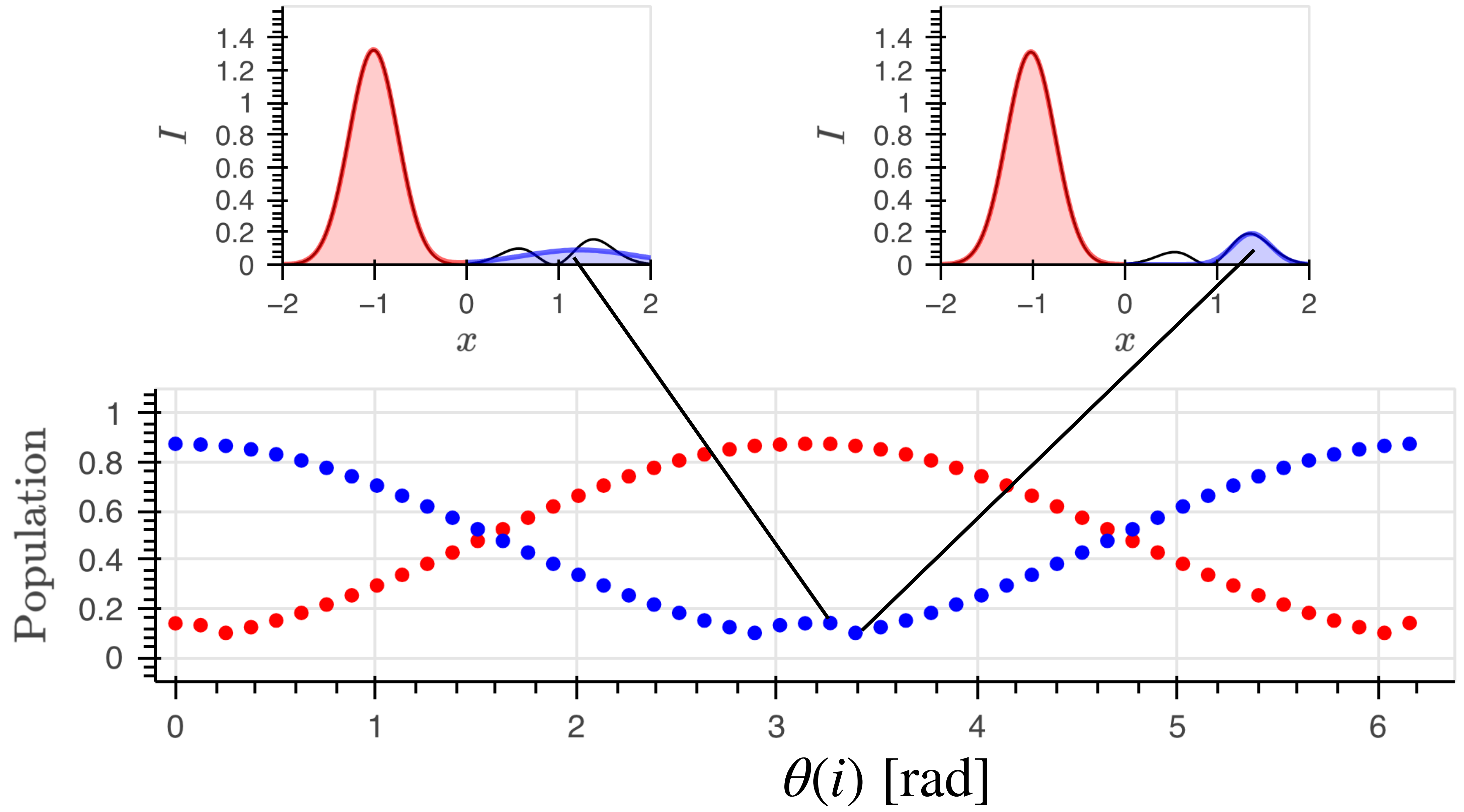

Signal Extraction with Aberrations

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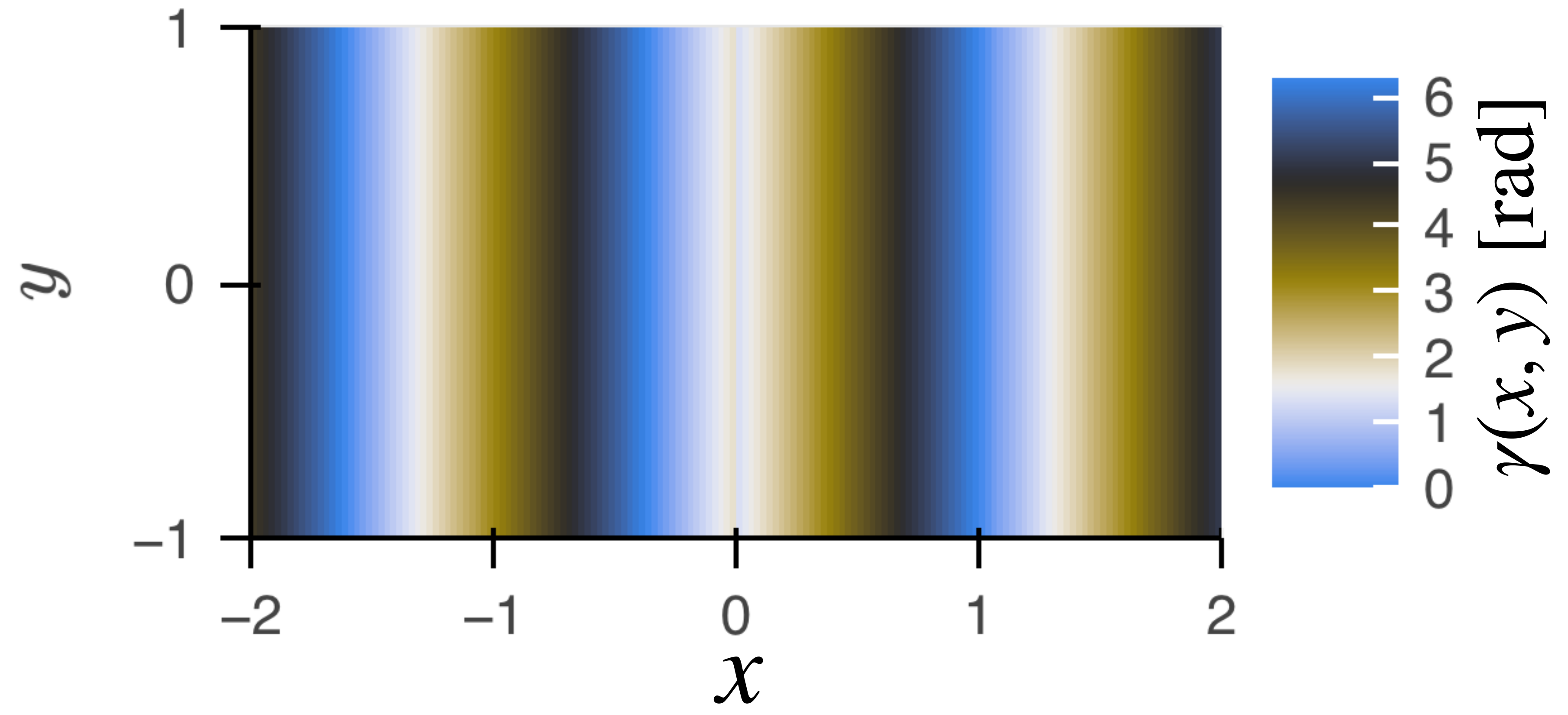

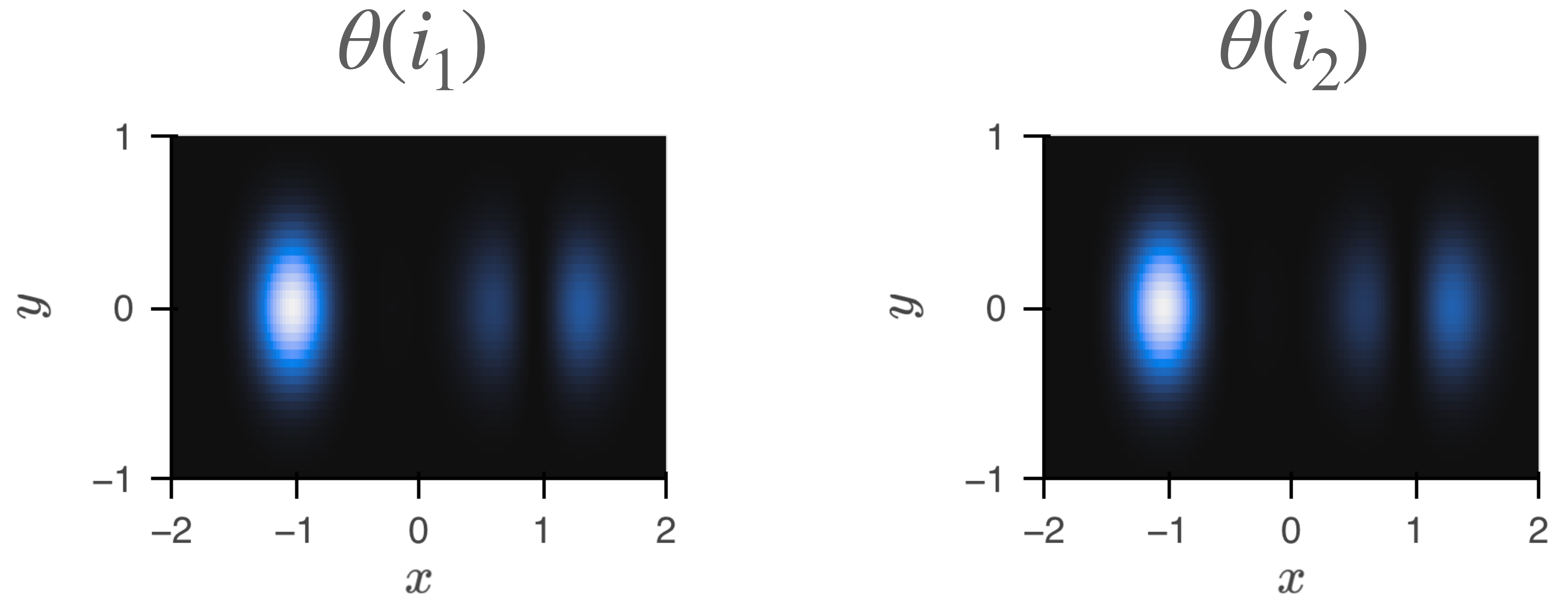

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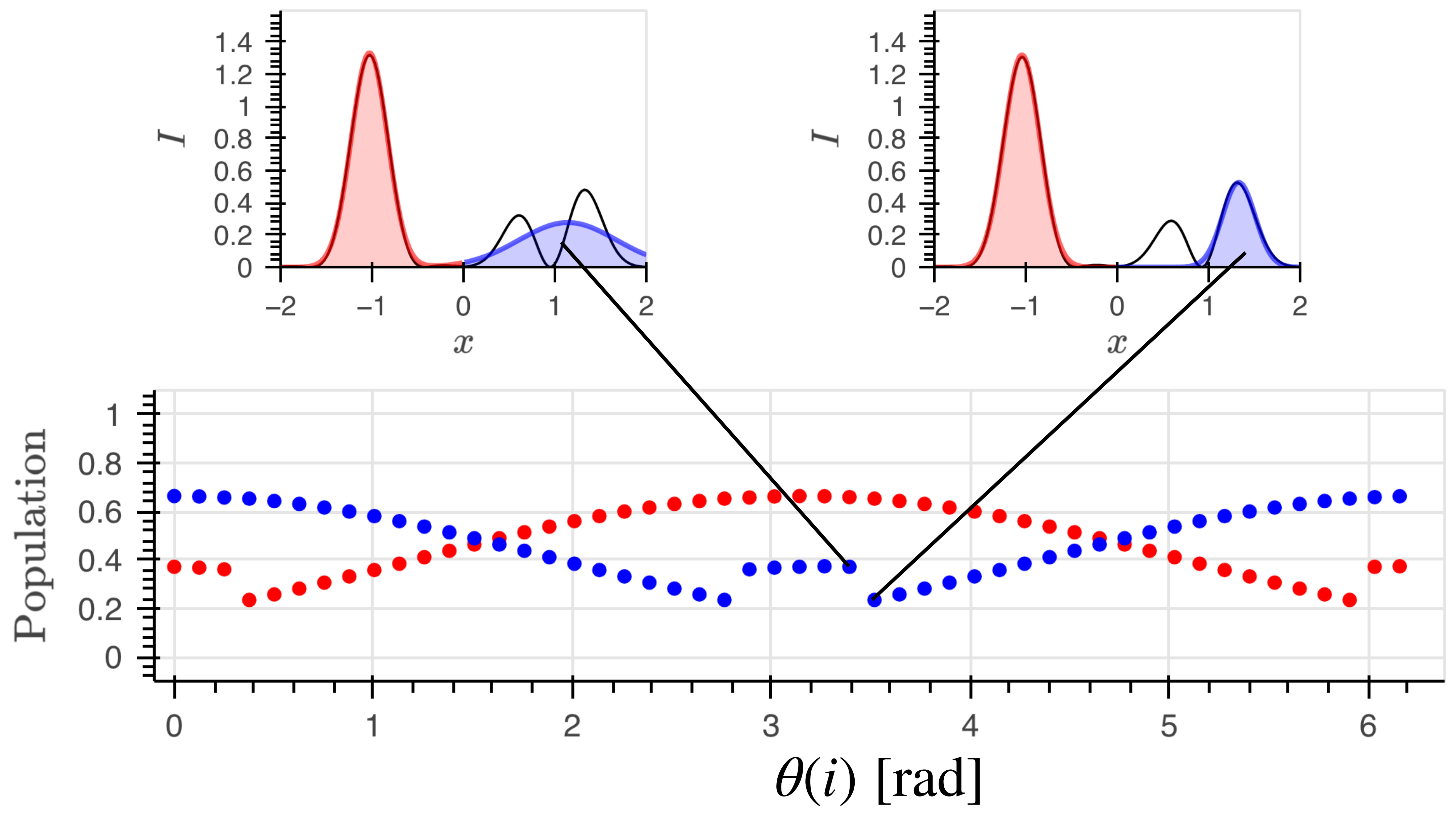

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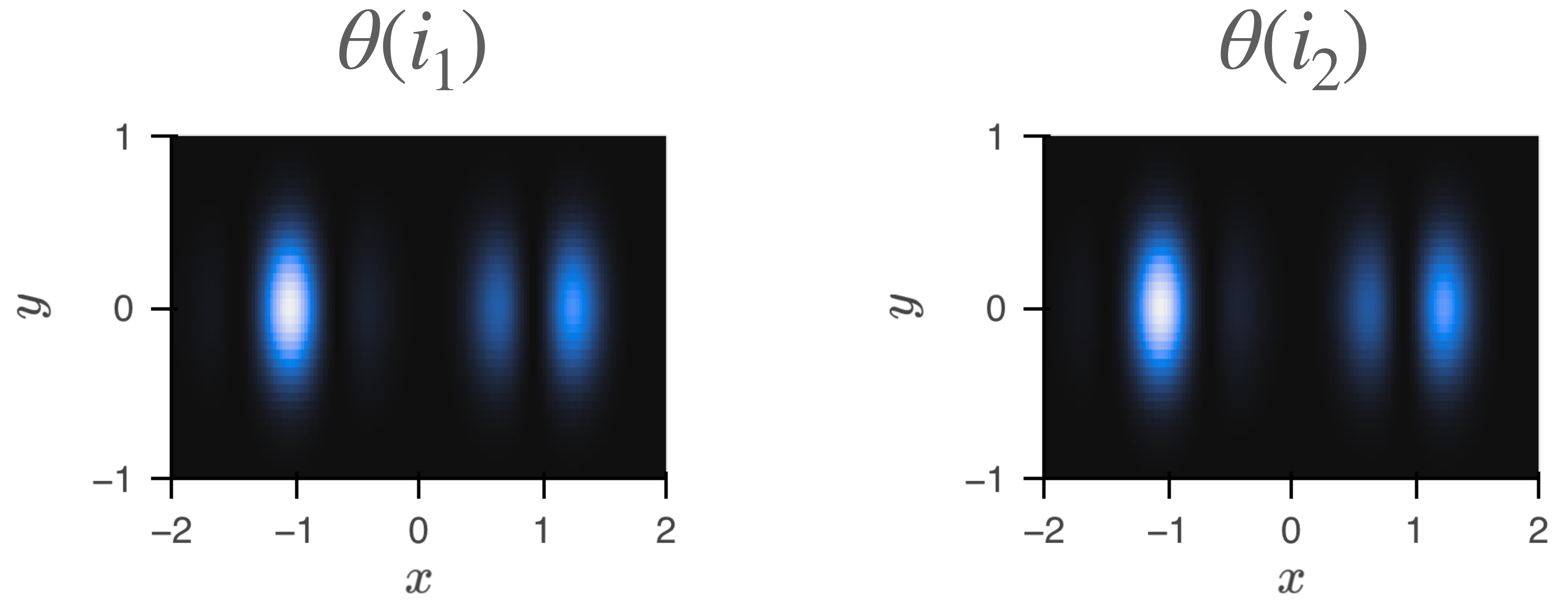

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$$I(i,x,y) = \frac{1}{2}\mathrm{env}(\mathrm{x},\mathrm{y})(1 + C\cos[\theta(i) + \gamma(x,y)])$$

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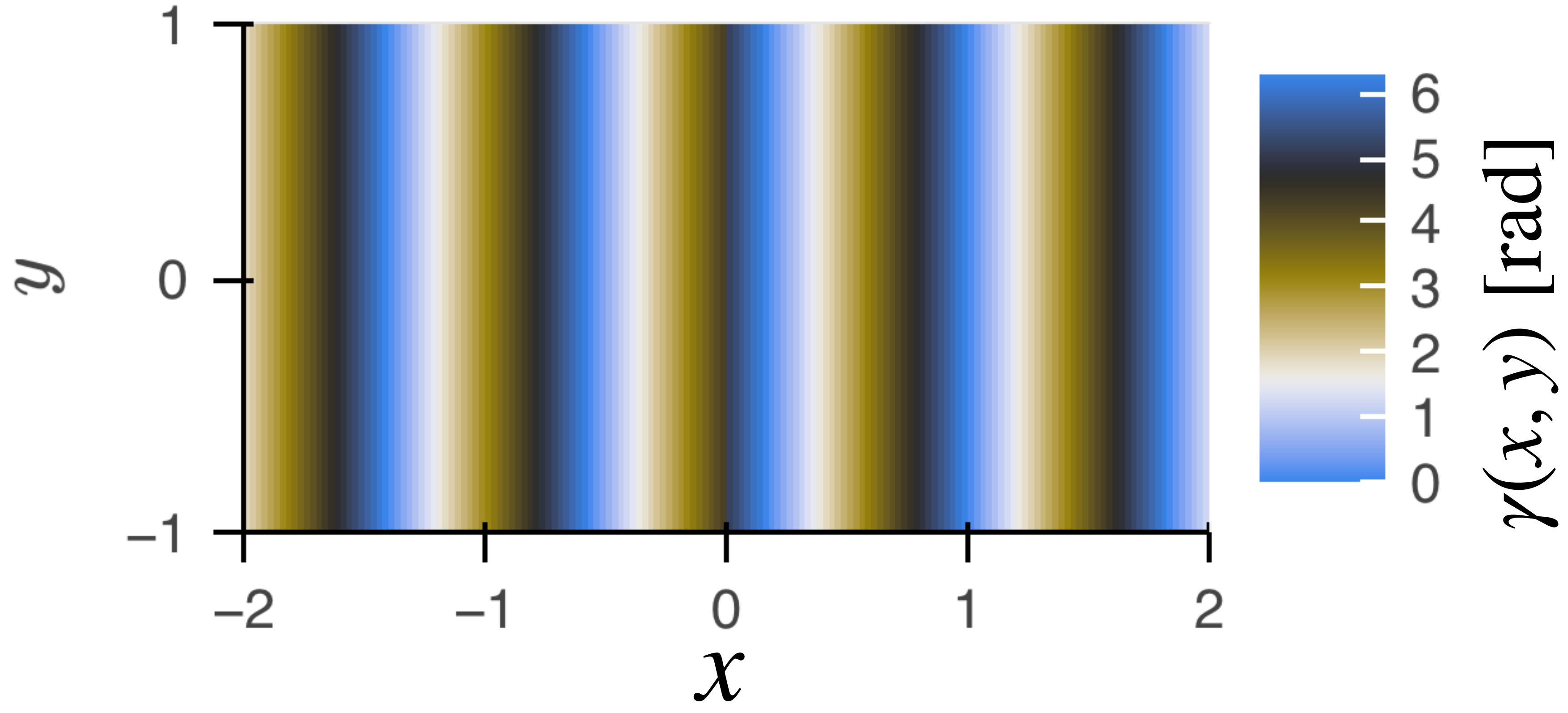
- Spatial phase

$$\gamma(x,y) = \begin{cases} 7.5(x-\delta x) & \text{if } x \geq 0 \\ 7.5(x+\delta x) + \pi & \text{if } x < 0 \end{cases}$$



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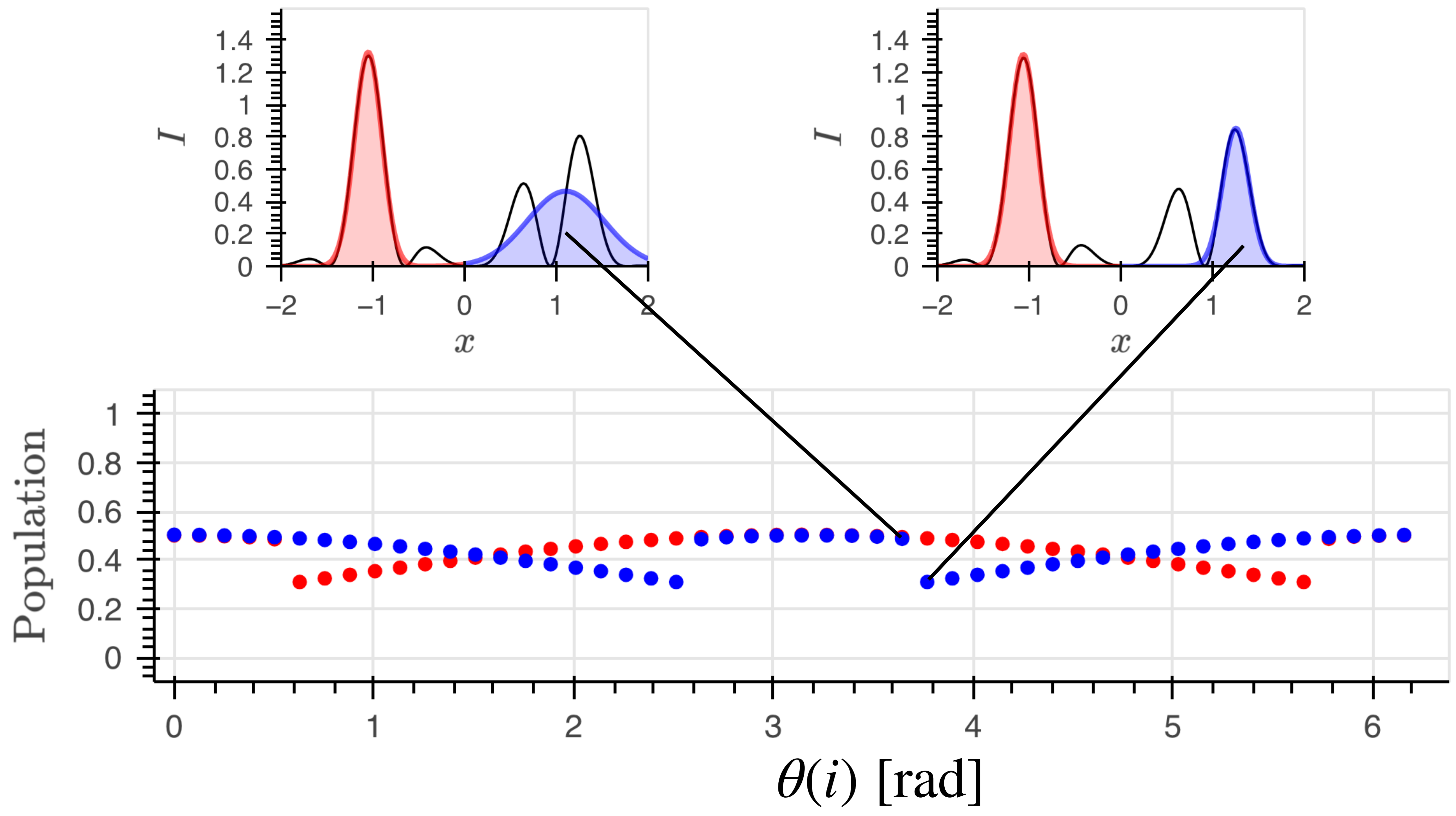

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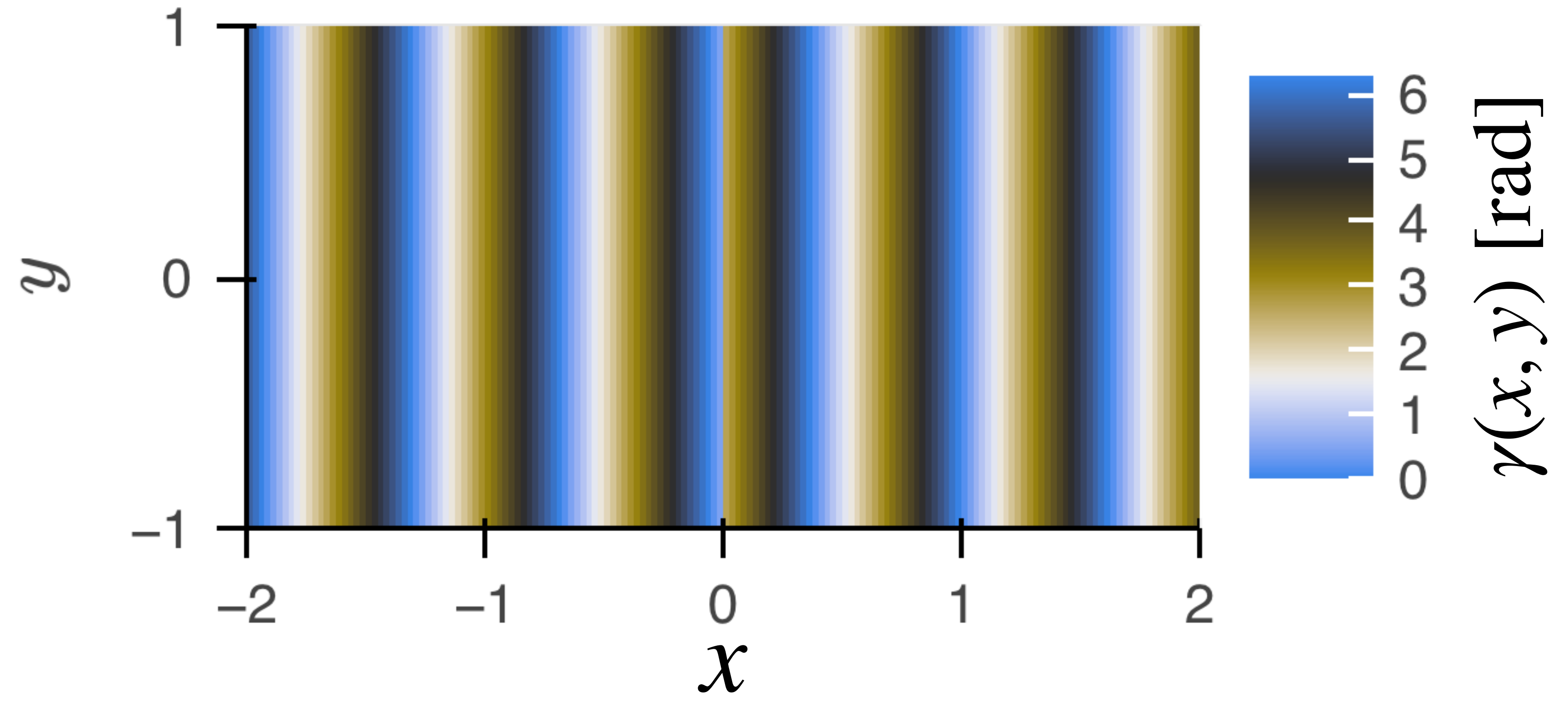
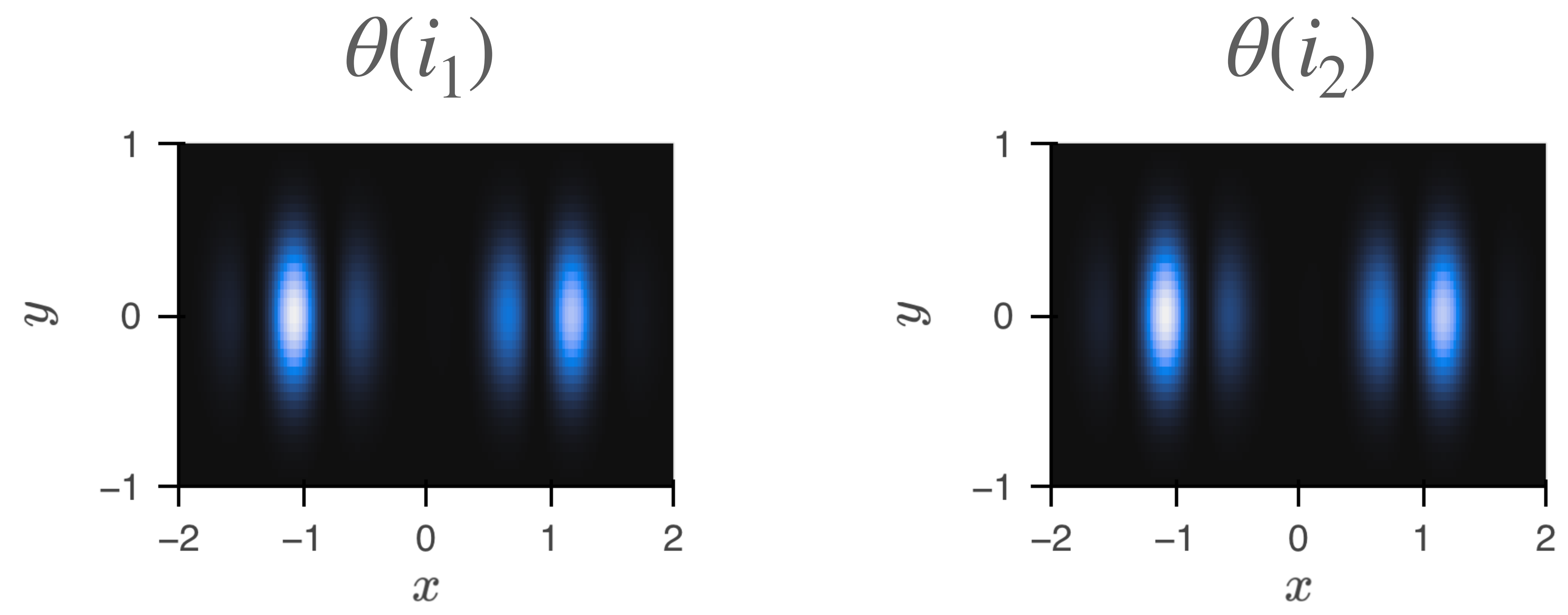
$$\theta(i) = \frac{2\pi i}{50} \text{ with } i \in [0,50) \quad \rightarrow f(\phi_a, \phi_L)$$

- Spatial phase

$$\gamma(x,y) = \begin{cases} 10.0(x - \delta x) & \text{if } x \geq 0 \\ 10.0(x + \delta x) + \pi & \text{if } x < 0 \end{cases}$$





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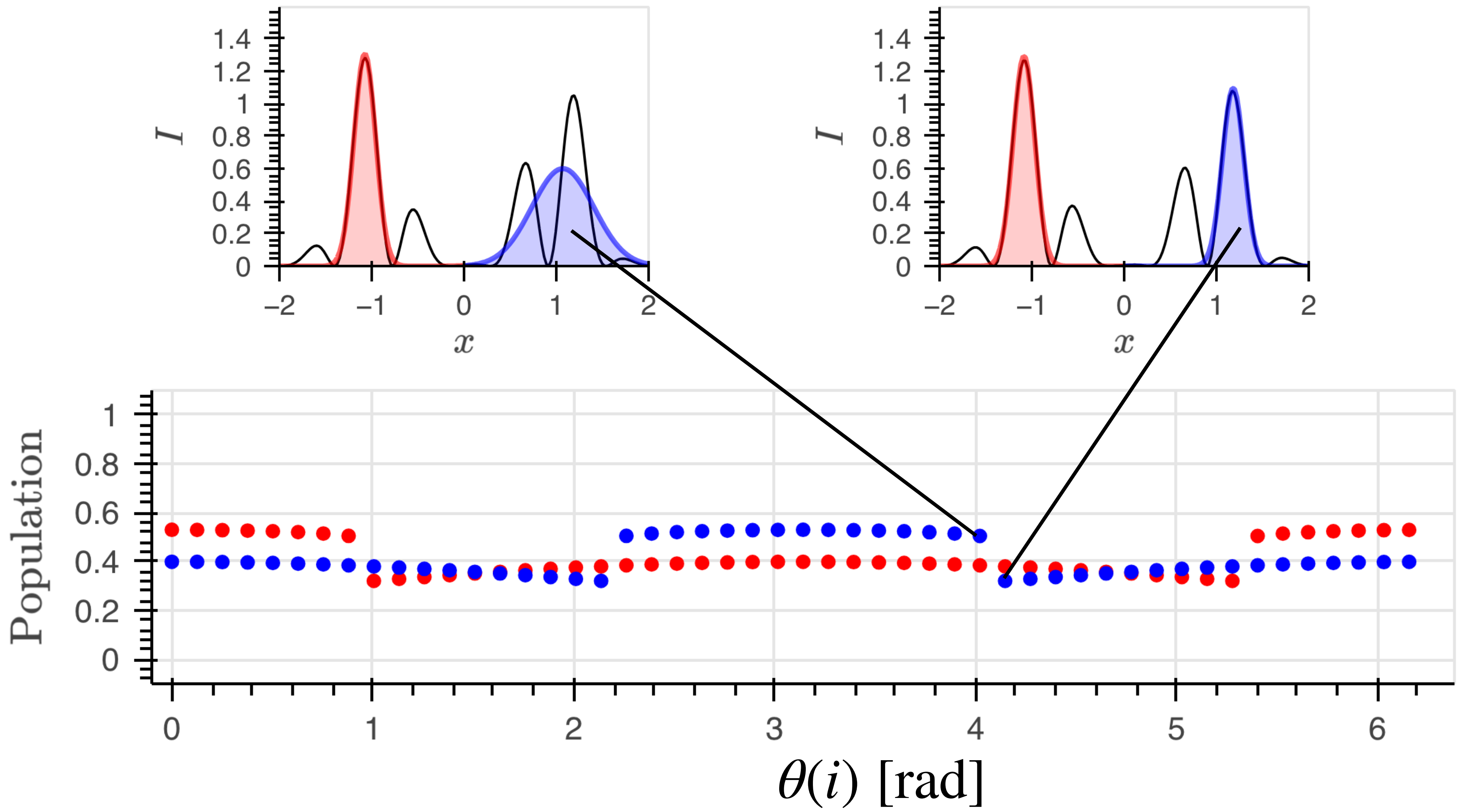
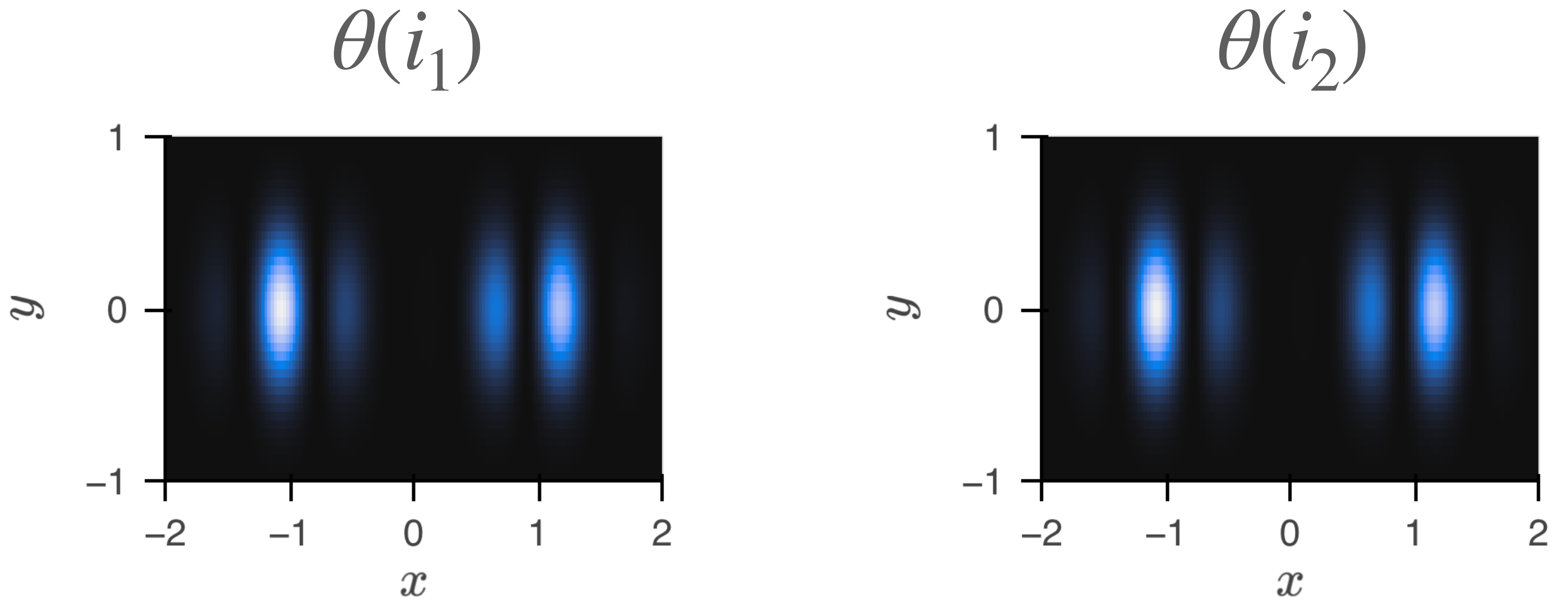


Signal Extraction with Aberrations

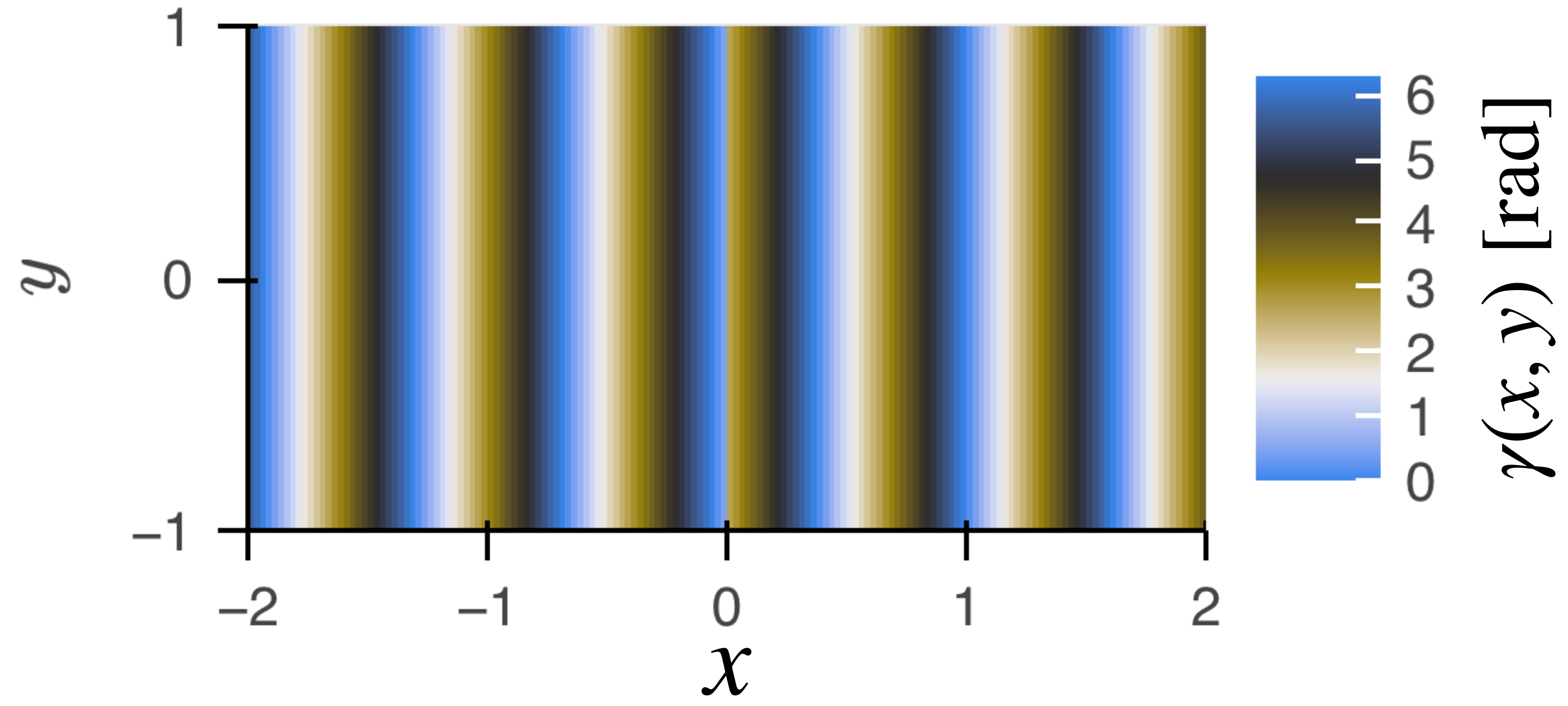


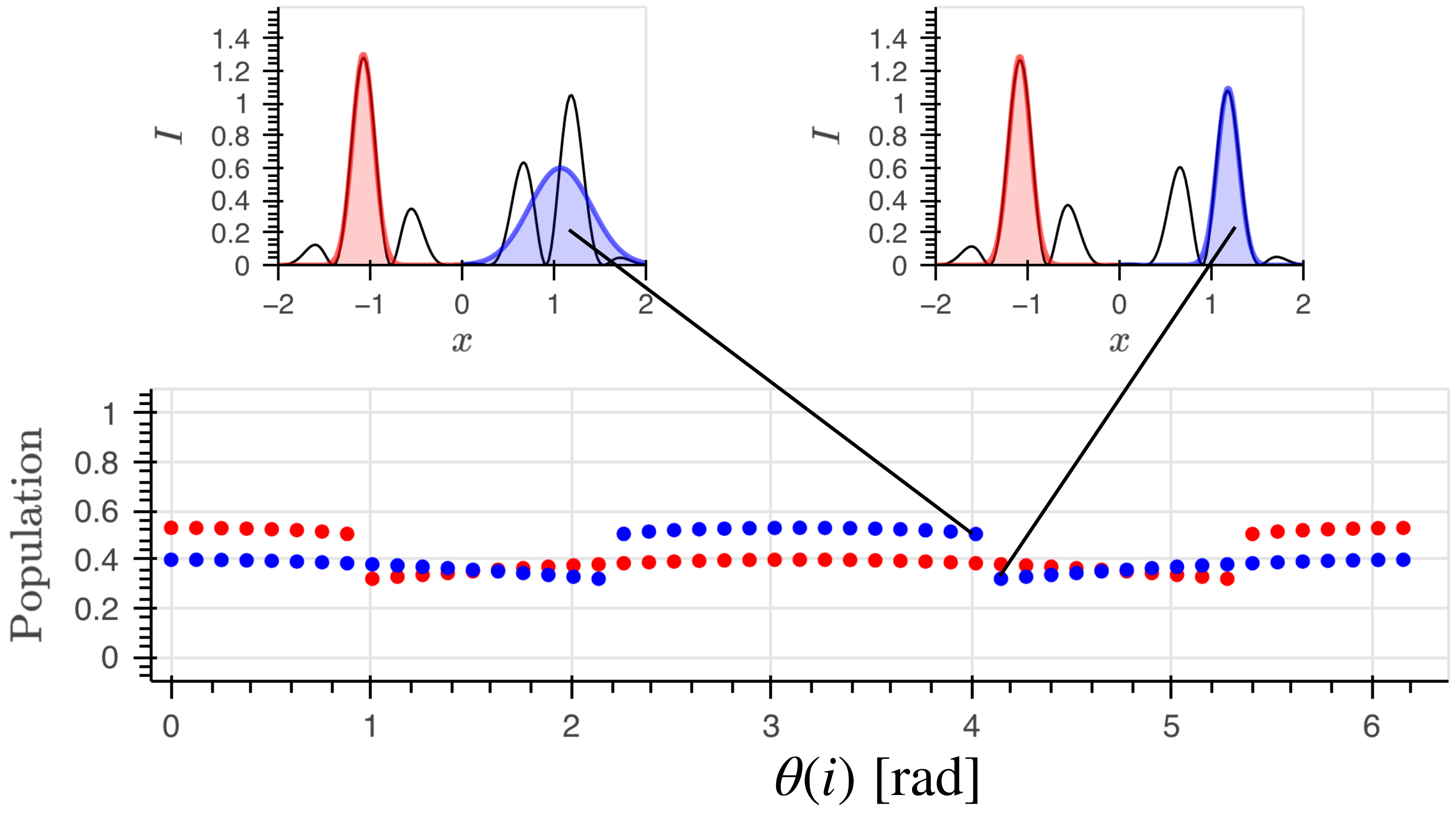
Adjust fitting model

- Allows single-shot phase scans [1,2]
- But requires knowledge of the spatial form

- Signal extraction via Gaussian integrals

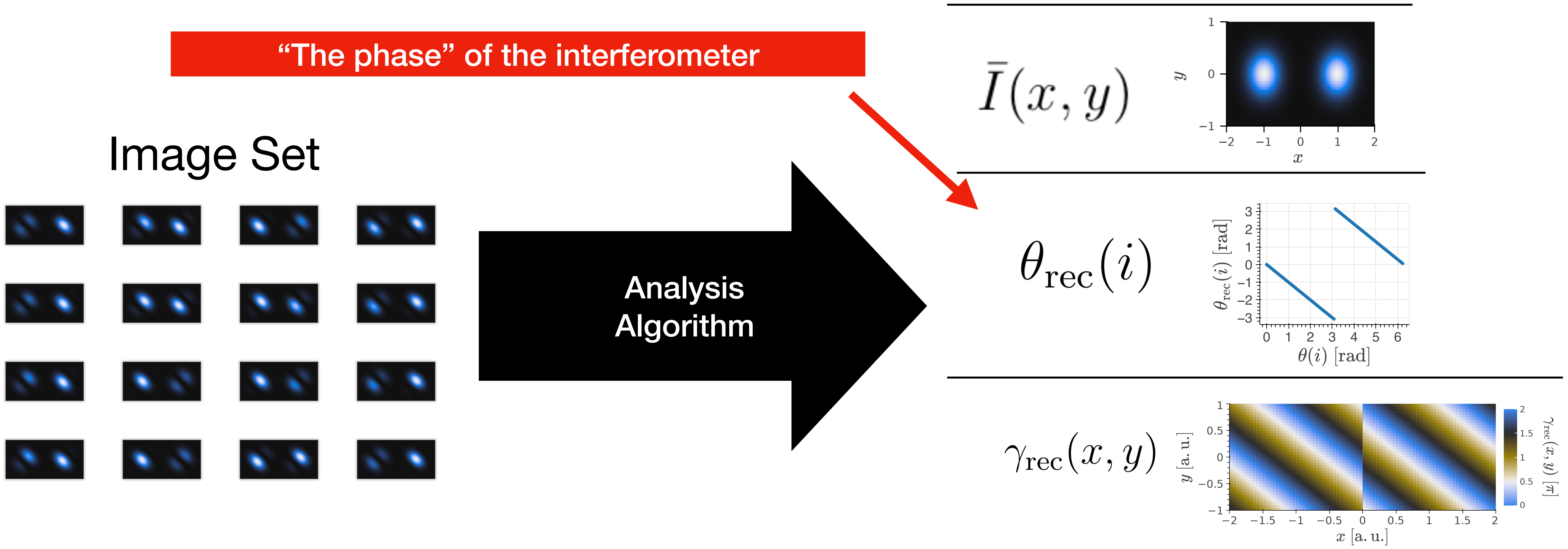
[1] Dickerson et al 2013 Phys. Rev. Lett. 111 083001
[2] Sugarbaker et al 2013 Phys. Rev. Lett. 111, 113002





Make Spatial Aberrations a Part of the Model

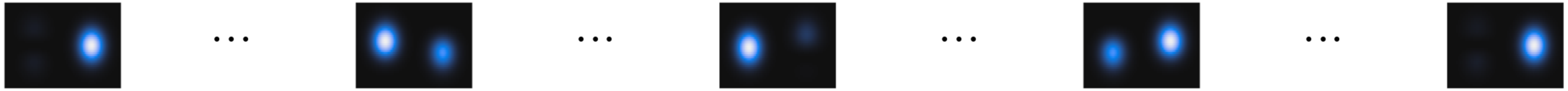
1. Use multiple images
2. Expand the image model to fit to ALL possible static patterns
3. Use statistics to extract the phases



Algorithm

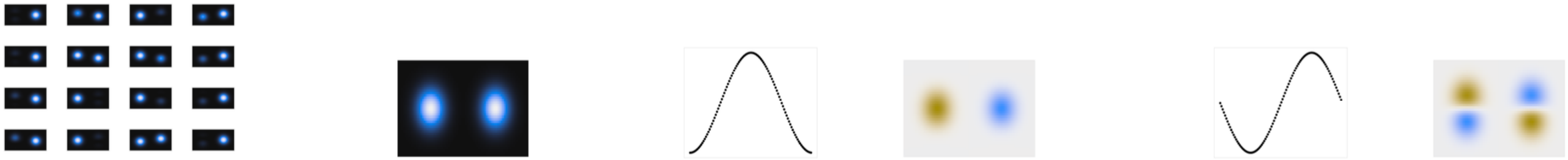
❶ Input Data

$I(0,x,y)$ … $I(30,x,y)$ … $I(60,x,y)$ … $I(80,x,y)$ … $I(99,x,y)$



❷ Principal component analysis (PCA)

$$I(i,x,y) = \bar{I}(x,y) + w_1(\imath)\ \mathrm{pc}_1(x,y) + w_2(\imath)\ \mathrm{pc}_2(x,y)$$



Algorithm

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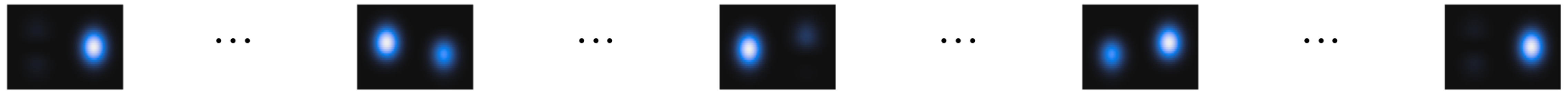
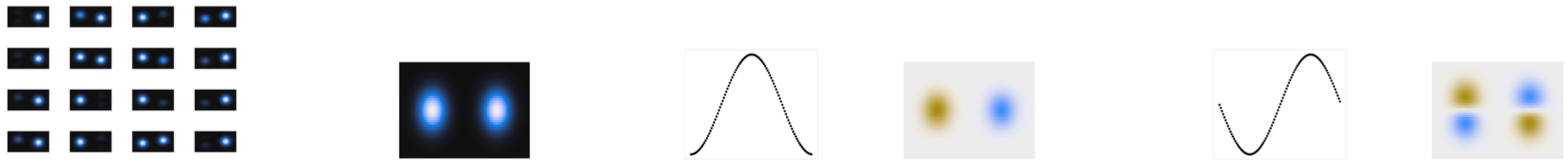
$I(0,x,y)$ … $I(30,x,y)$ … $I(60,x,y)$ … $I(80,x,y)$ … $I(99,x,y)$

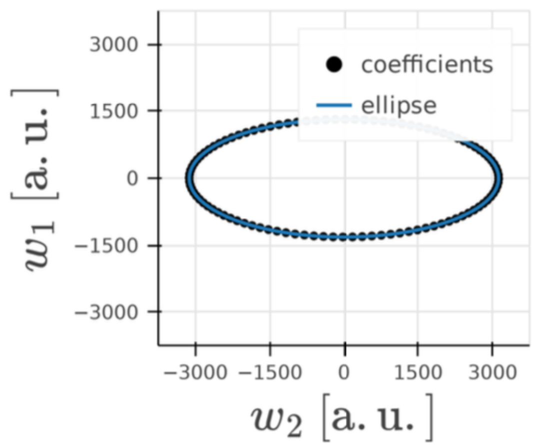


❷ PCA

$$I(i,x,y) = \bar{I}(x,y) + w_1(i)\ \mathrm{pc}_1(x,y) + w_2(i)\ \mathrm{pc}_2(x,y)$$



❸ Ellipse Fit



$$w_1(i) = x_c + a\cos\vartheta\cos t_i - b\sin\vartheta\sin t_i$$
$$-w_2(i) = y_c + a\sin\vartheta\cos t_i + b\cos\vartheta\sin t_i$$

❹ Ellipse Correction

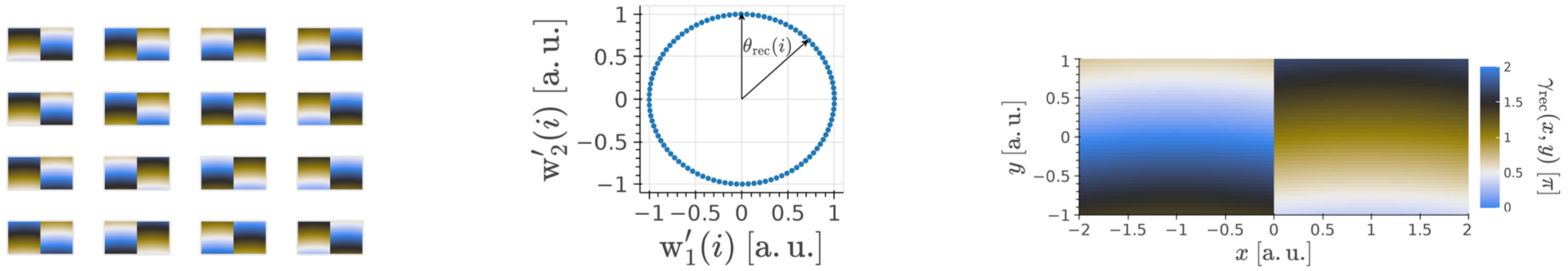
$$\begin{pmatrix} w'_1(i) \\ w'_2(i) \end{pmatrix} = \underbrace{\begin{pmatrix} 1/a & 0 \\ 0 & 1/b \end{pmatrix}}_{\text{scale}} \times \underbrace{\begin{pmatrix} \cos(-\vartheta) & -\sin(-\vartheta) \\ \sin(-\vartheta) & \cos(-\vartheta) \end{pmatrix}}_{\text{rotate}} \times \left[\begin{pmatrix} w_1(i) \\ -w_2(i) \end{pmatrix} \underbrace{- \begin{pmatrix} x_c \\ y_c \end{pmatrix}}_{\text{translate}}\right]$$

$$\begin{pmatrix} \mathrm{pc}'_1(x,y) \\ \mathrm{pc}'_2(x,y) \end{pmatrix} = \underbrace{\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}}_{\text{scale}} \times \underbrace{\begin{pmatrix} \cos(\vartheta) & -\sin(\vartheta) \\ \sin(\vartheta) & \cos(\vartheta) \end{pmatrix}}_{\text{rotate}} \times \begin{pmatrix} \mathrm{pc}_1(x,y) \\ \mathrm{pc}_2(x,y) \end{pmatrix}$$

❺ Phase Reconstruction

$$\theta_{\mathrm{rec}}(i) = \mathrm{arctan2}(w'_2(i), w'_1(i))$$
$$\gamma_{\mathrm{rec}}(x,y) = \mathrm{arctan2}(\mathrm{pc}'_2(x,y), \mathrm{pc}'_1(x,y))$$

$$\Phi_{\mathrm{rec}}(i,x,y) = \theta_{\mathrm{rec}}(i) + \gamma_{\mathrm{rec}}(x,y)$$



Reconstruction Without Noise

- Intensity model

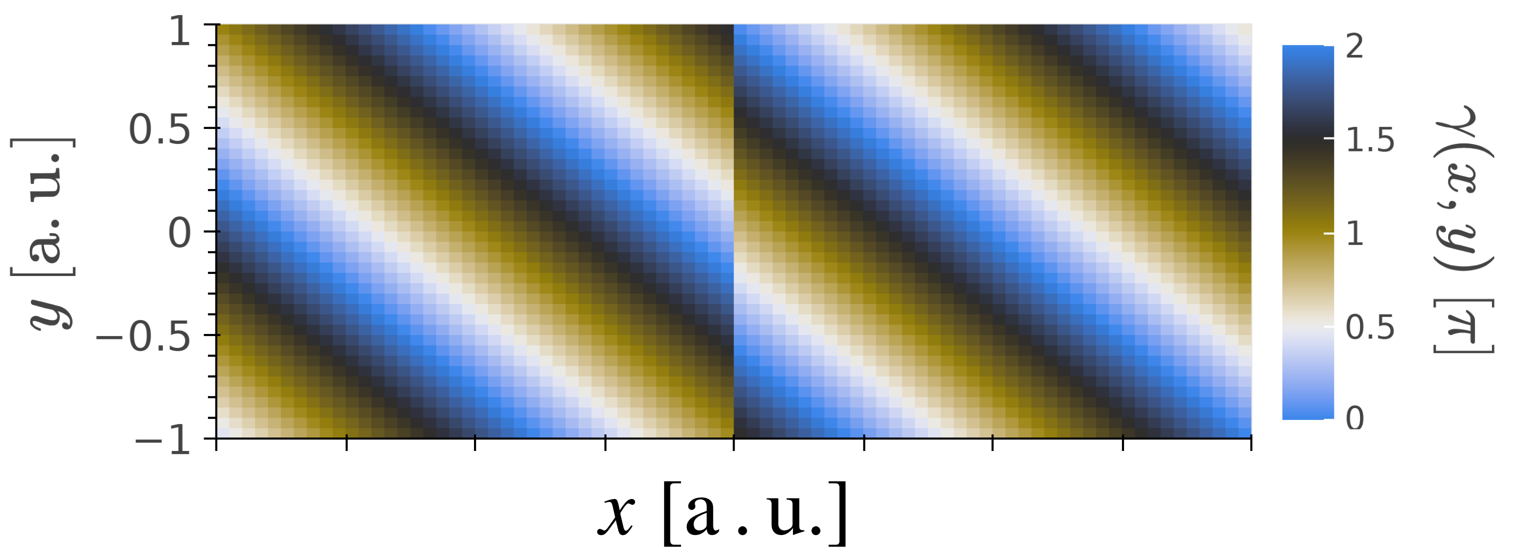
$$I(i,x,y) = \frac{1}{2}\mathrm{env(x,y)}(1 + C\cos[\theta(i) + \gamma(x,y)])$$

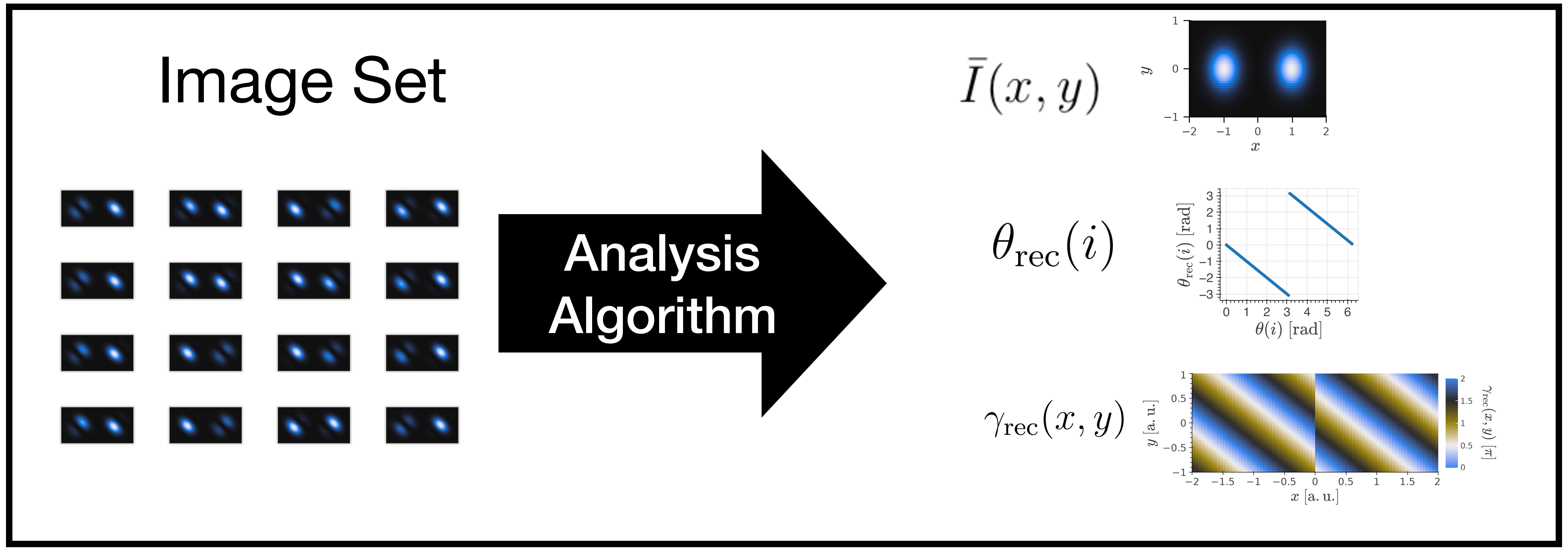
- Per image phase

$$\theta(i) = \frac{2\pi i}{50} \text{ with } i \in [0,50)$$

- Spatial phase

$$\gamma(x,y) = \begin{cases} 4(x-\delta x) + 4y & \text{if } x \geq 0 \\ 4(x+\delta x) + 4y + \pi & \text{if } x < 0 \end{cases}$$





Reconstruction Without Noise

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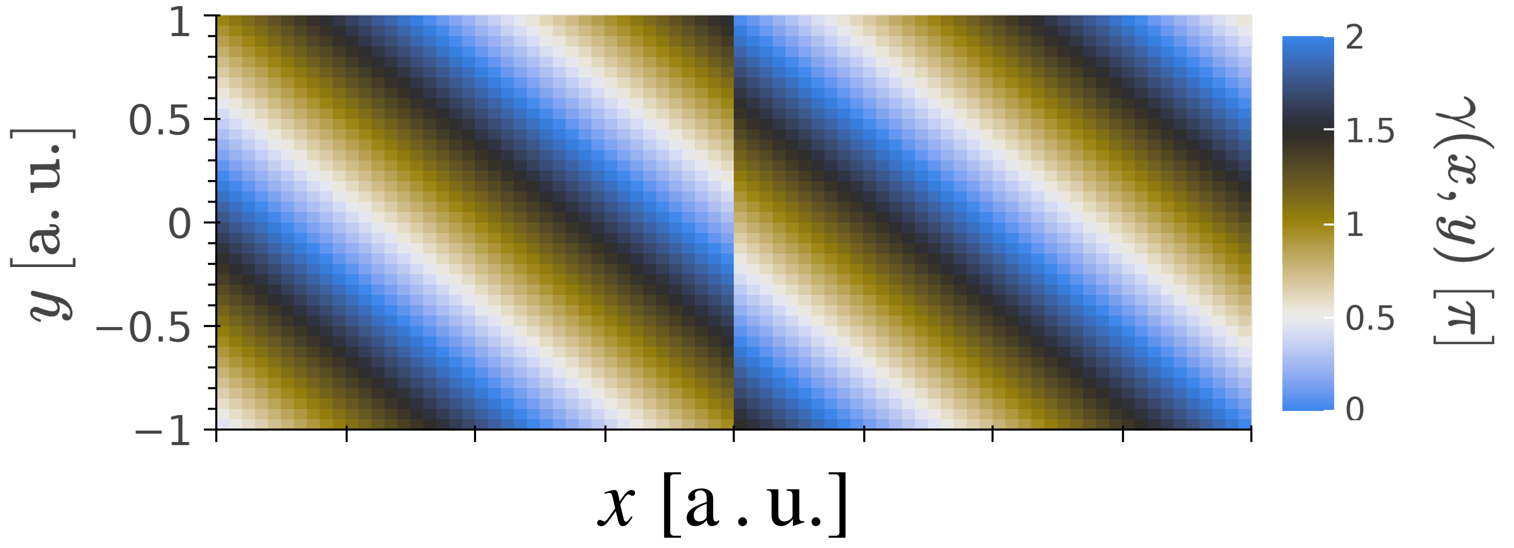
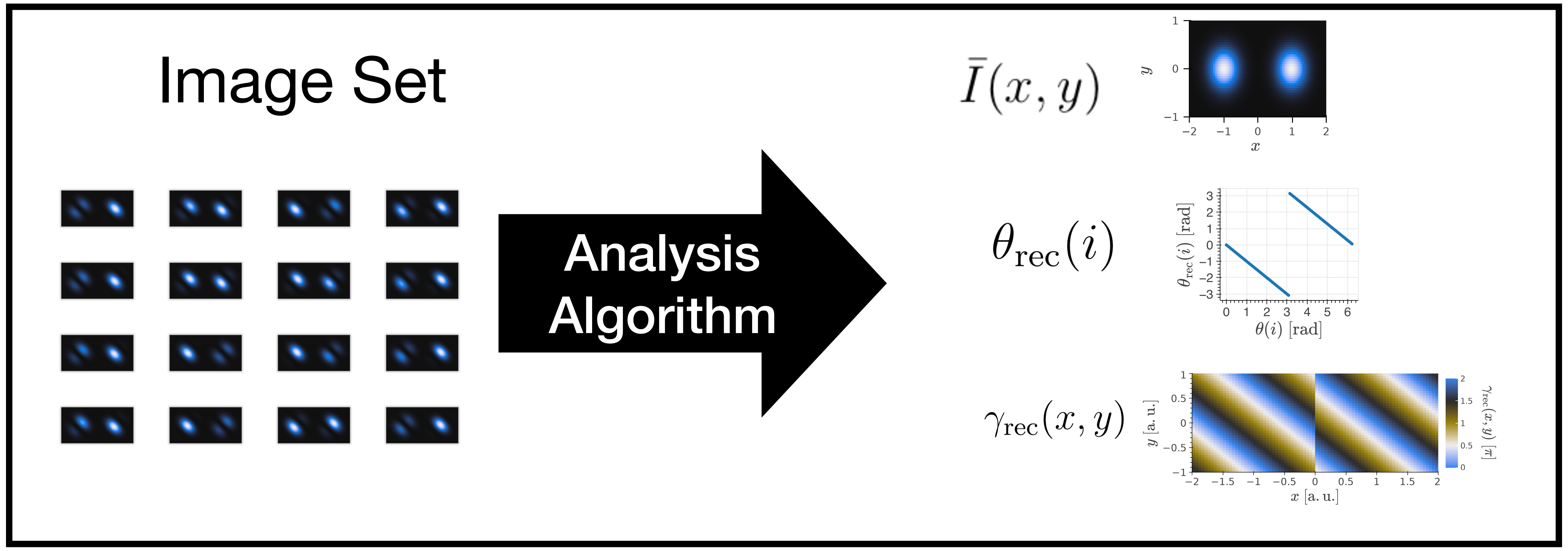
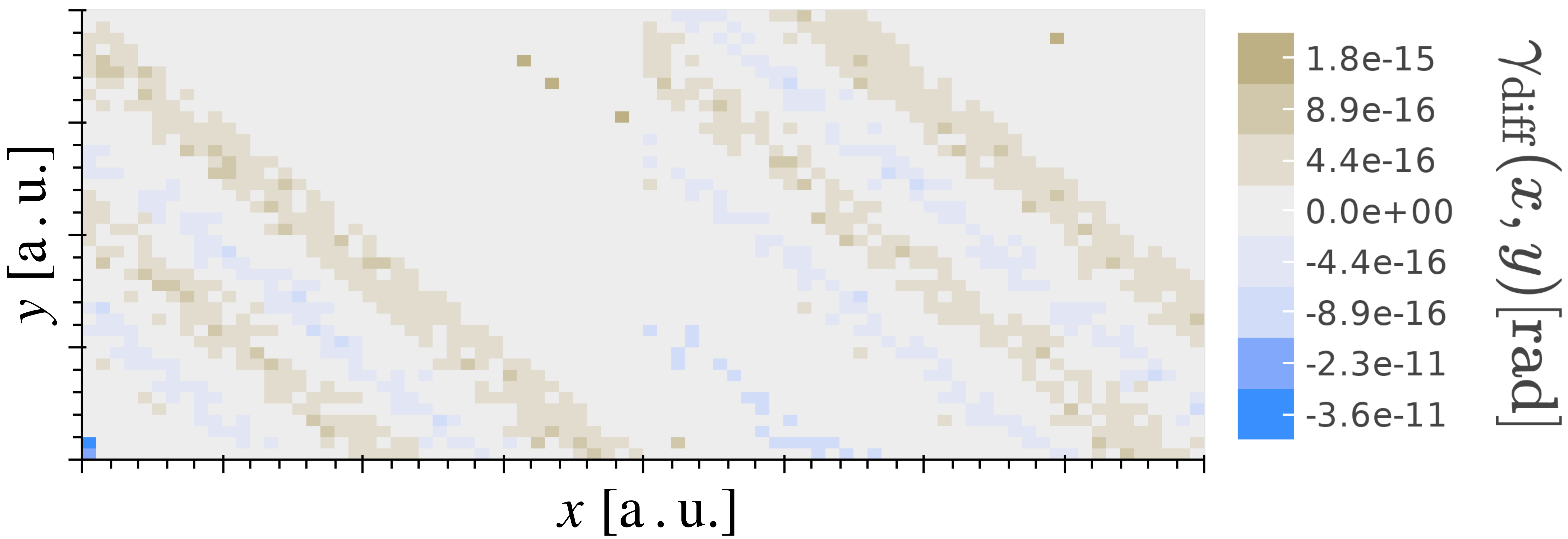


Works perfectly!



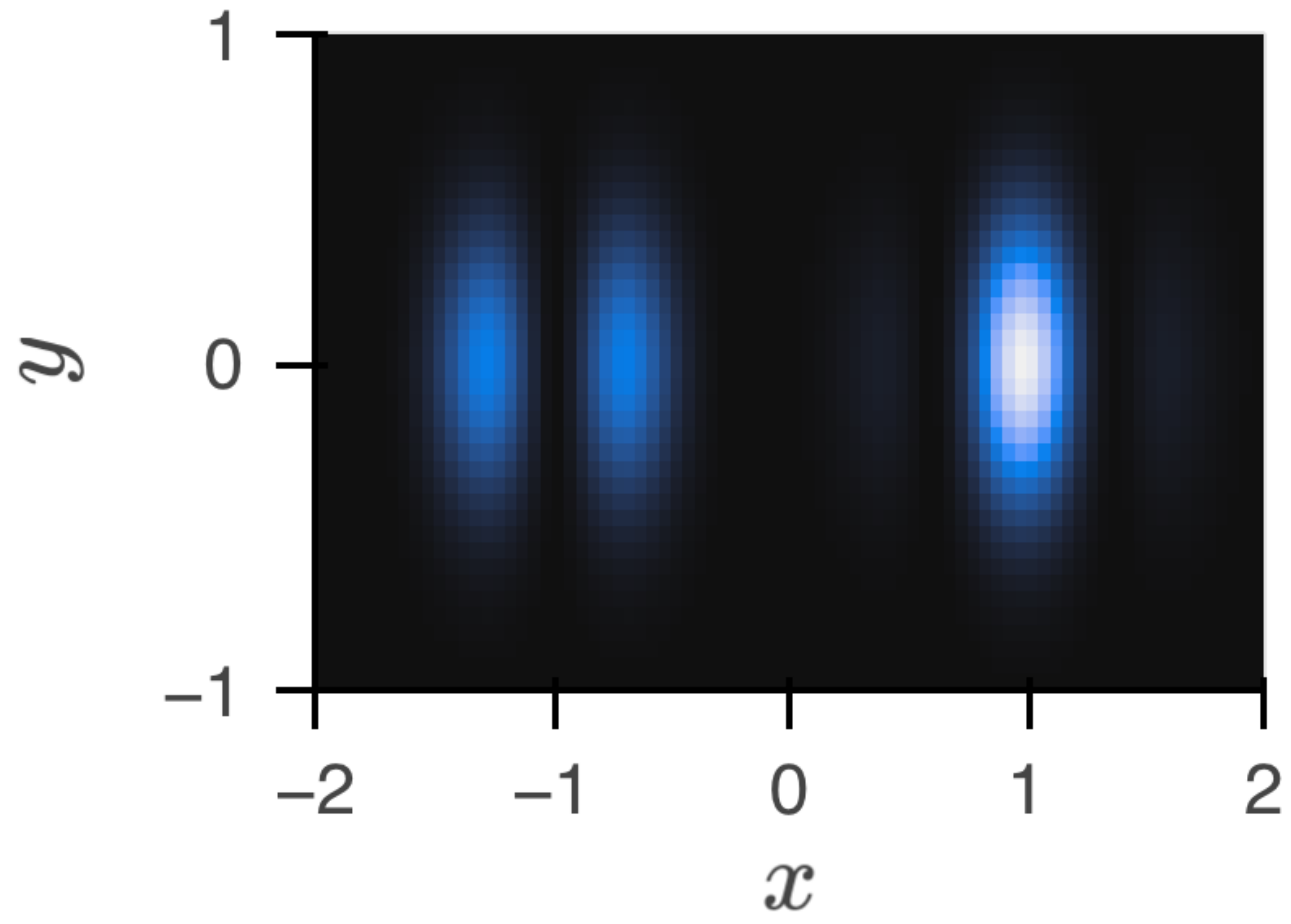
Reconstruction error $\gamma_{\mathrm{diff}} = \gamma_{\mathrm{inp}} - \gamma_{\mathrm{rec}}$



Quantum Projection Noise (QPN)

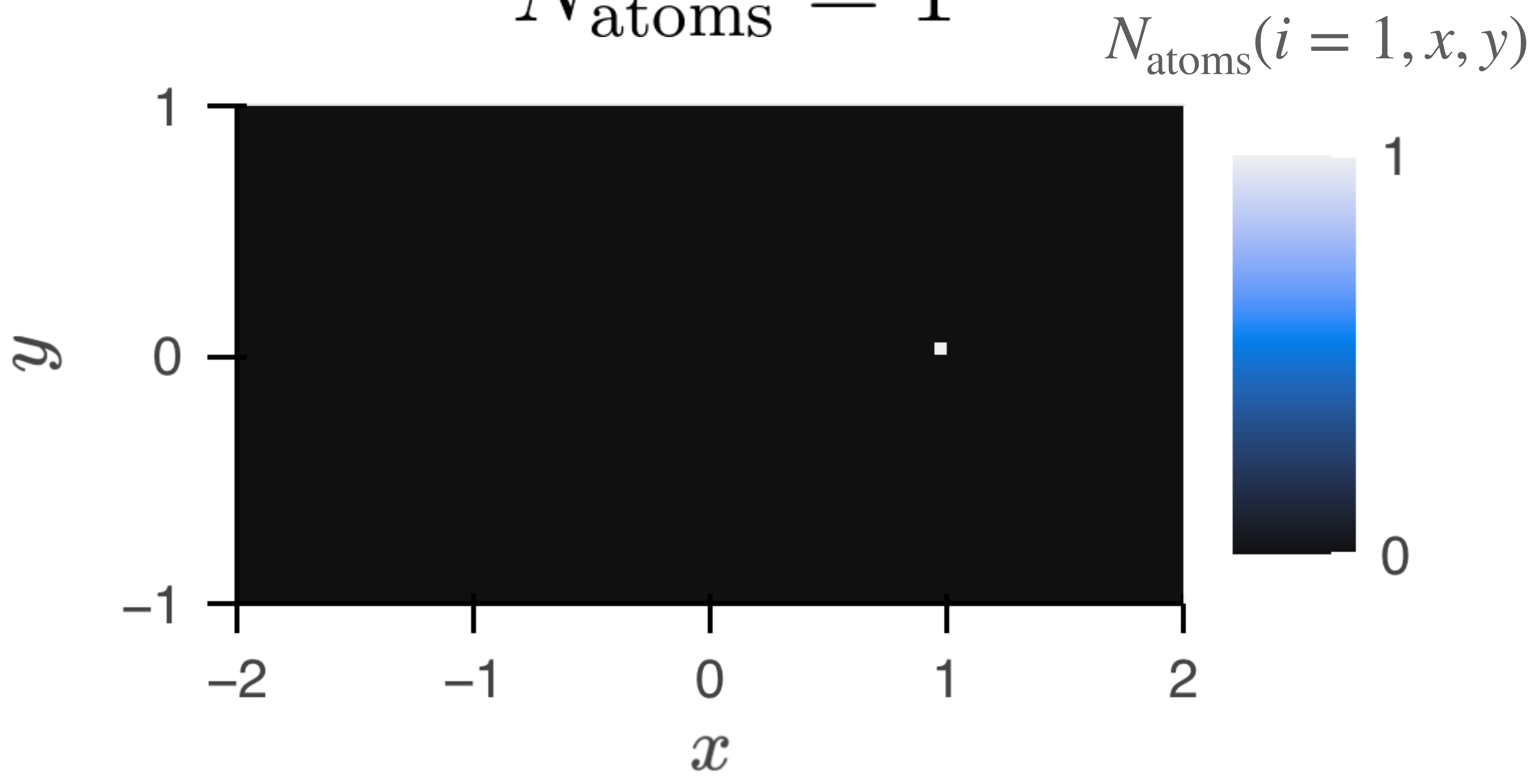
Spatial probability distribution

$$P(x, y)$$



Interferogram

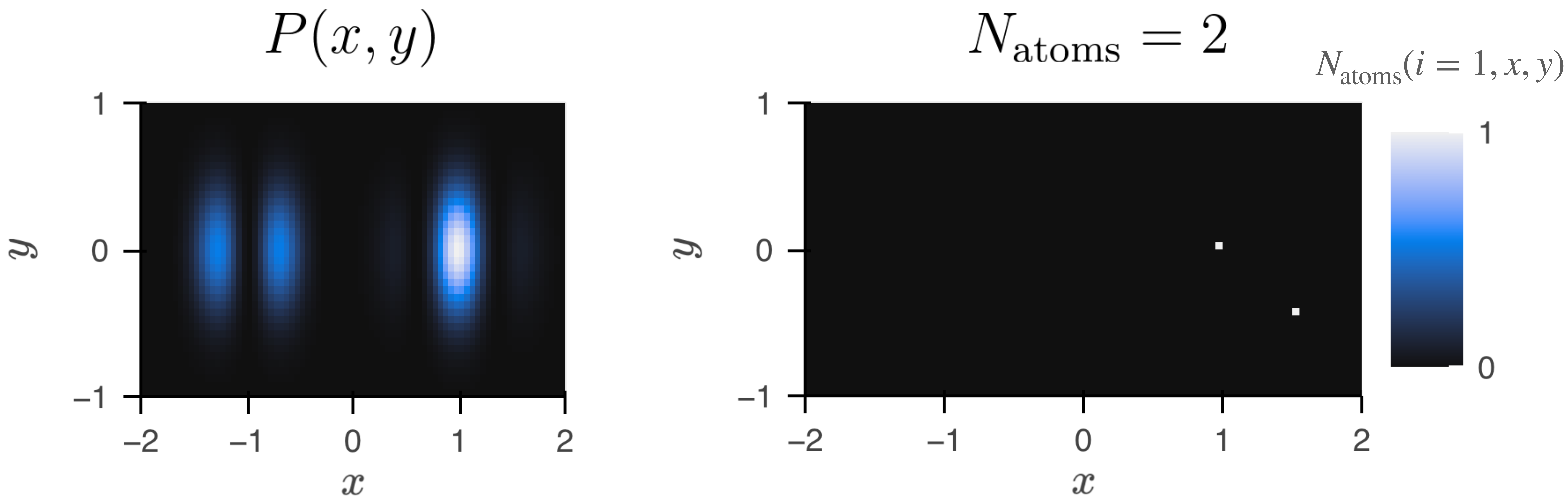
$$N_{\mathrm{atoms}} = 1$$



Quantum Projection Noise (QPN)

Spatial probability distribution

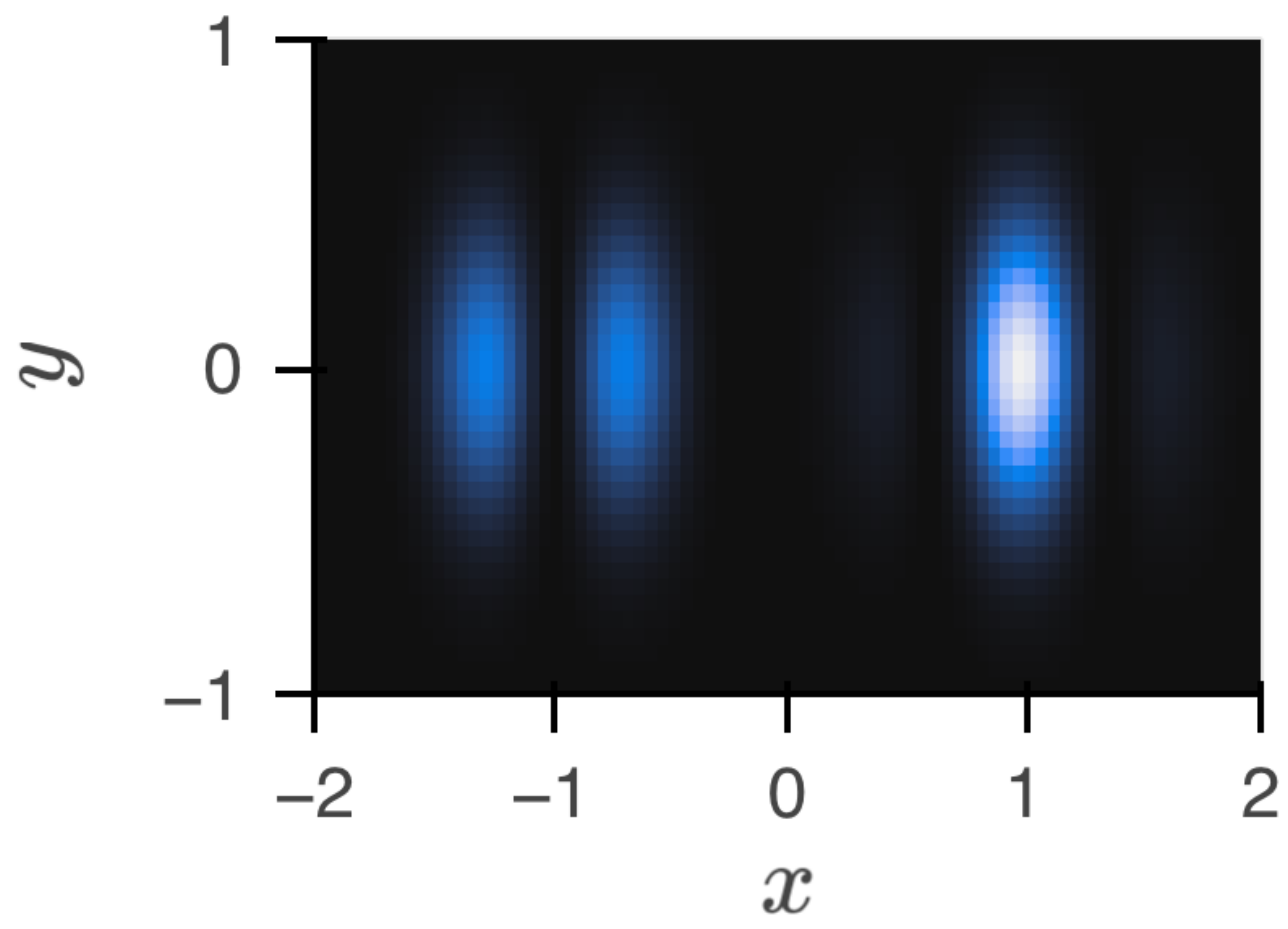
Interferogram



Quantum Projection Noise (QPN)

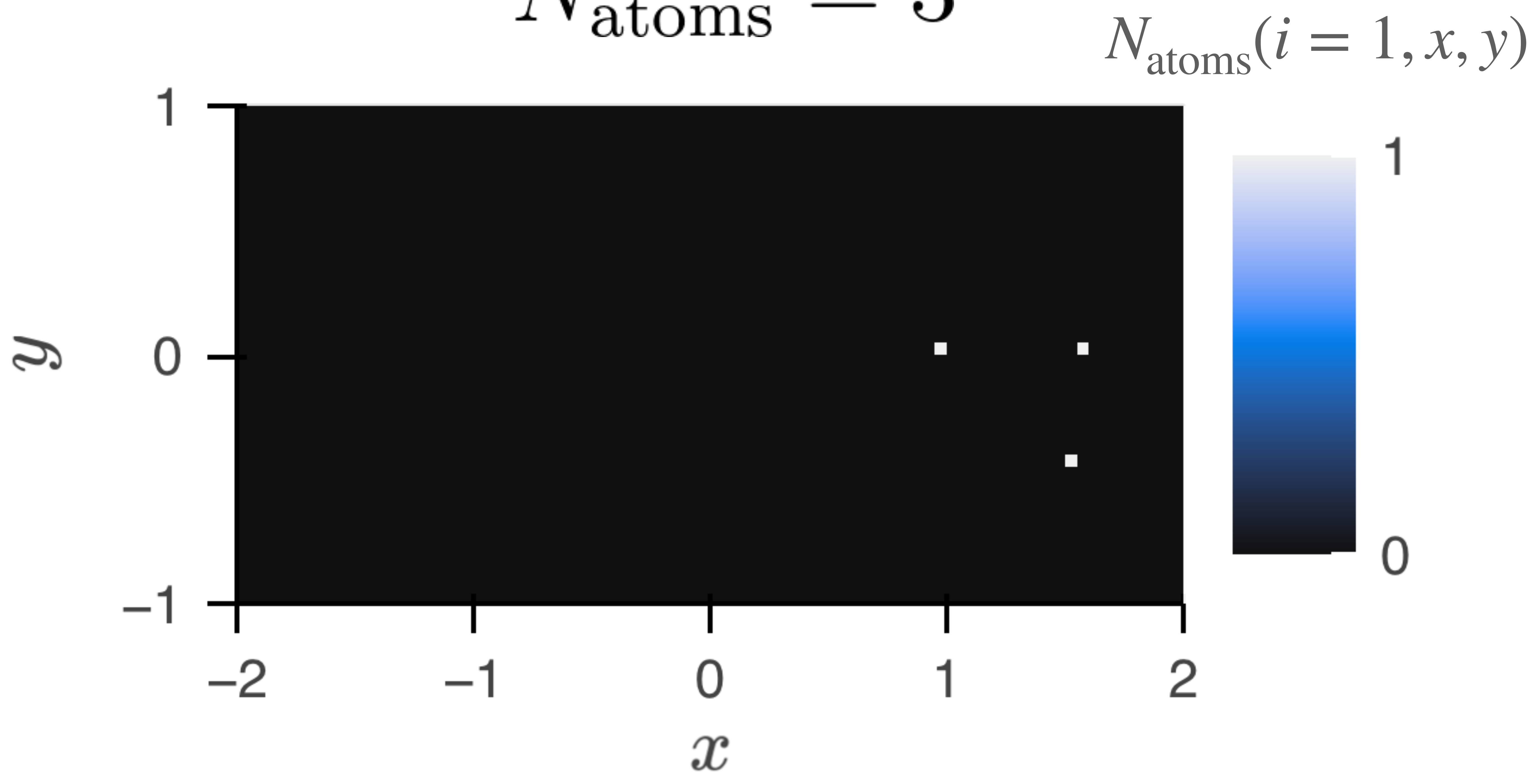
Spatial probability distribution

$$P(x, y)$$



Interferogram

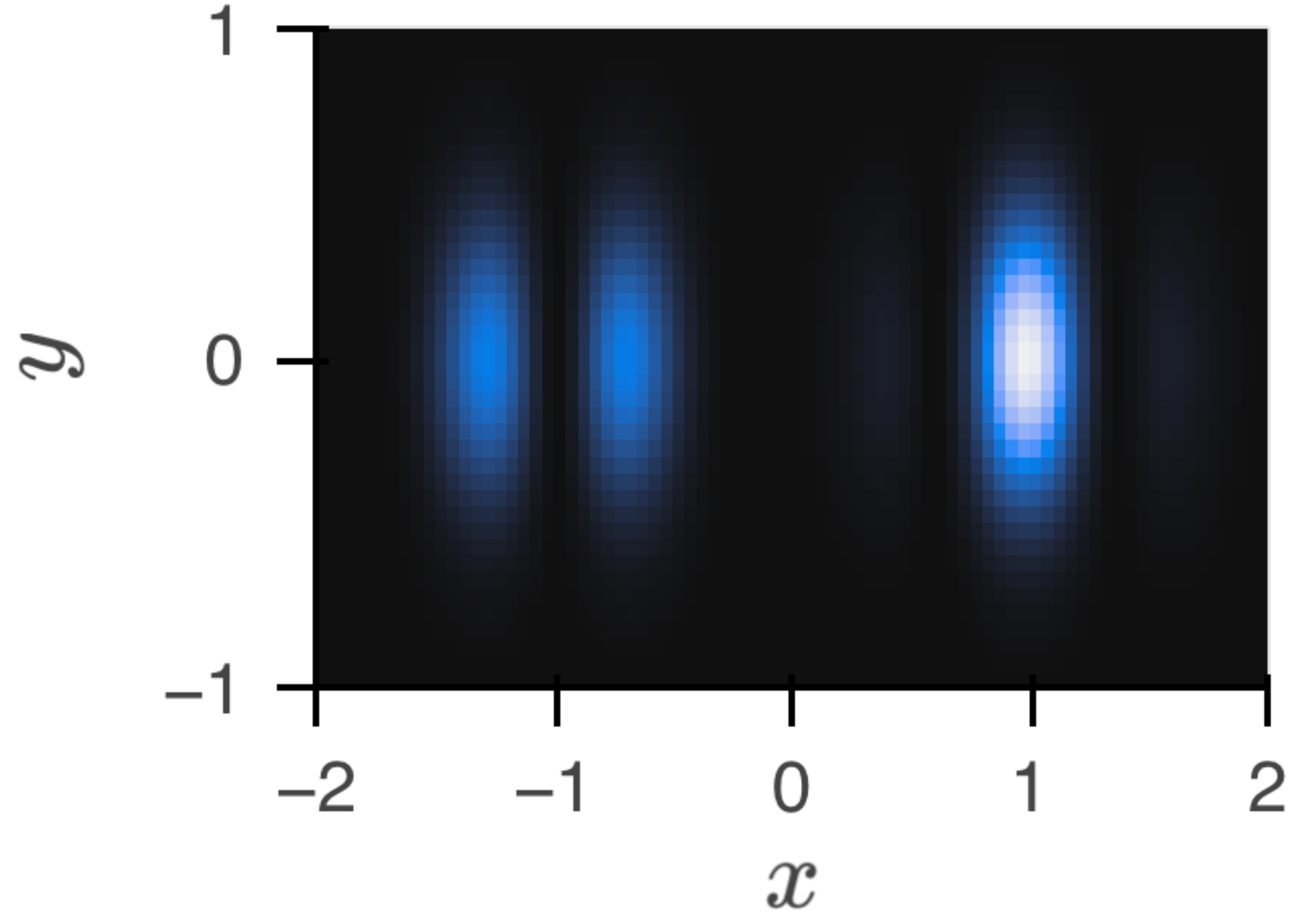
$$N_{\text{atoms}} = 3$$



Quantum Projection Noise (QPN)

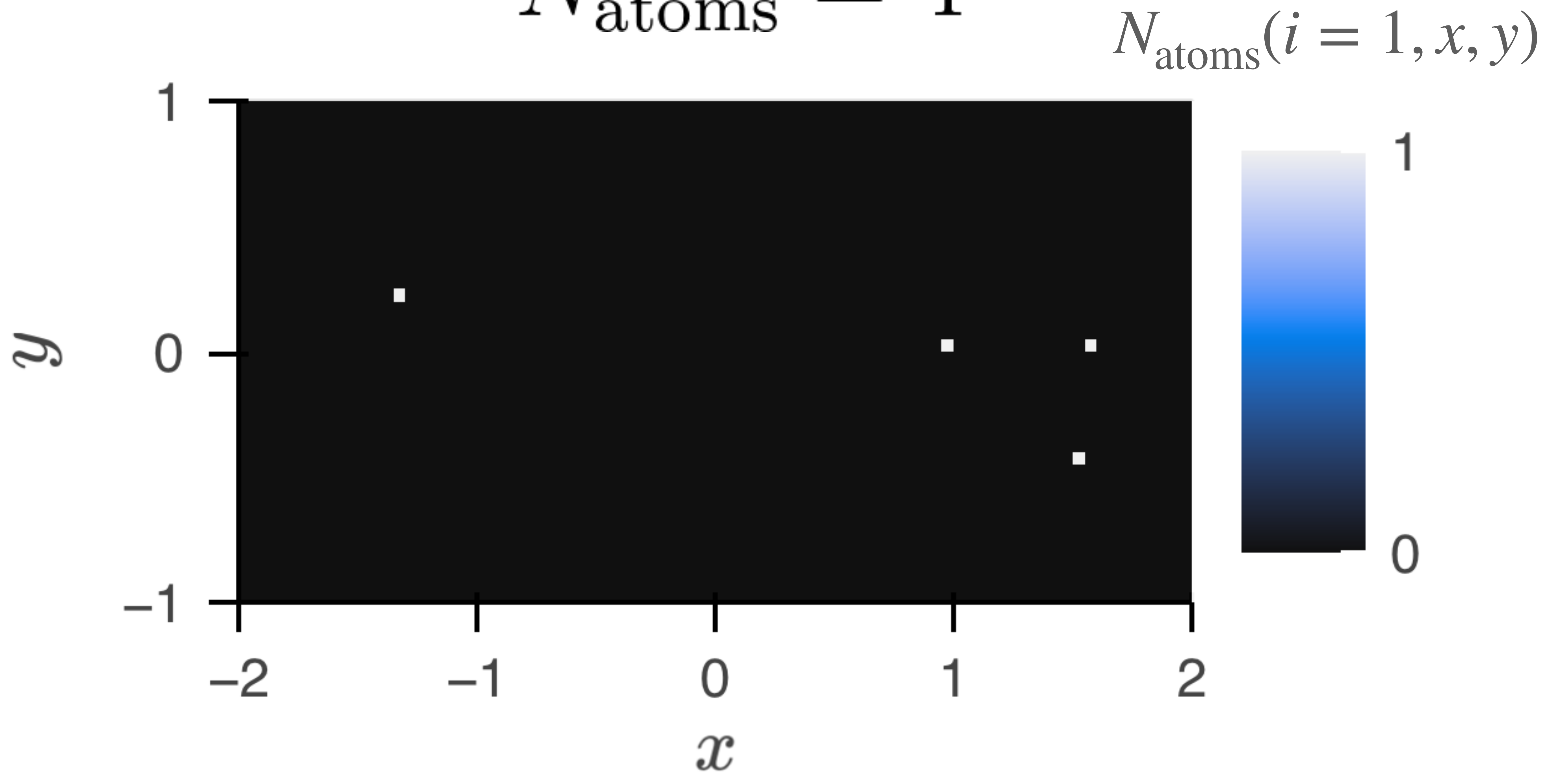
Spatial probability distribution

$$P(x, y)$$



Interferogram

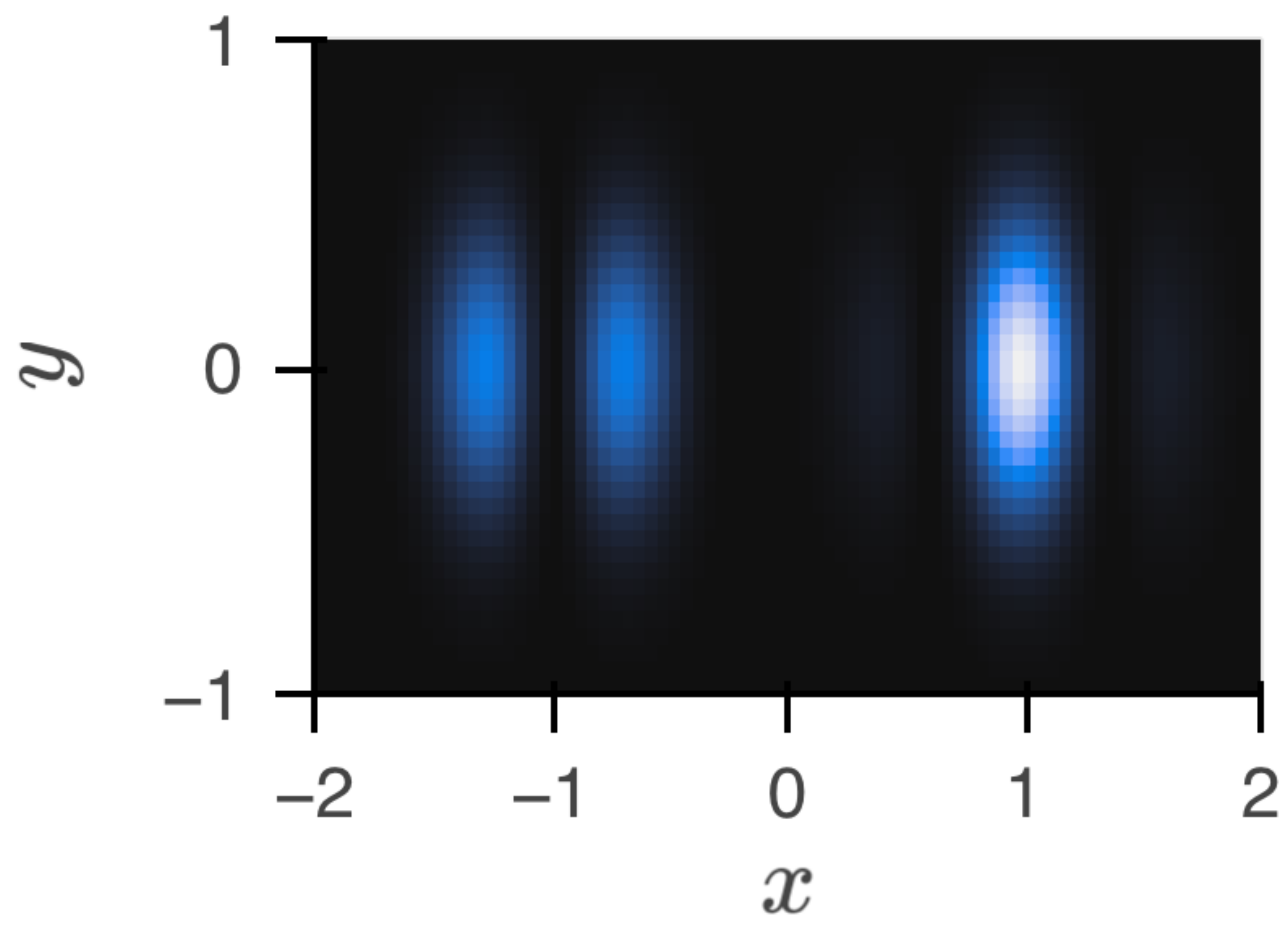
$$N_{\text{atoms}} = 4$$



Quantum Projection Noise (QPN)

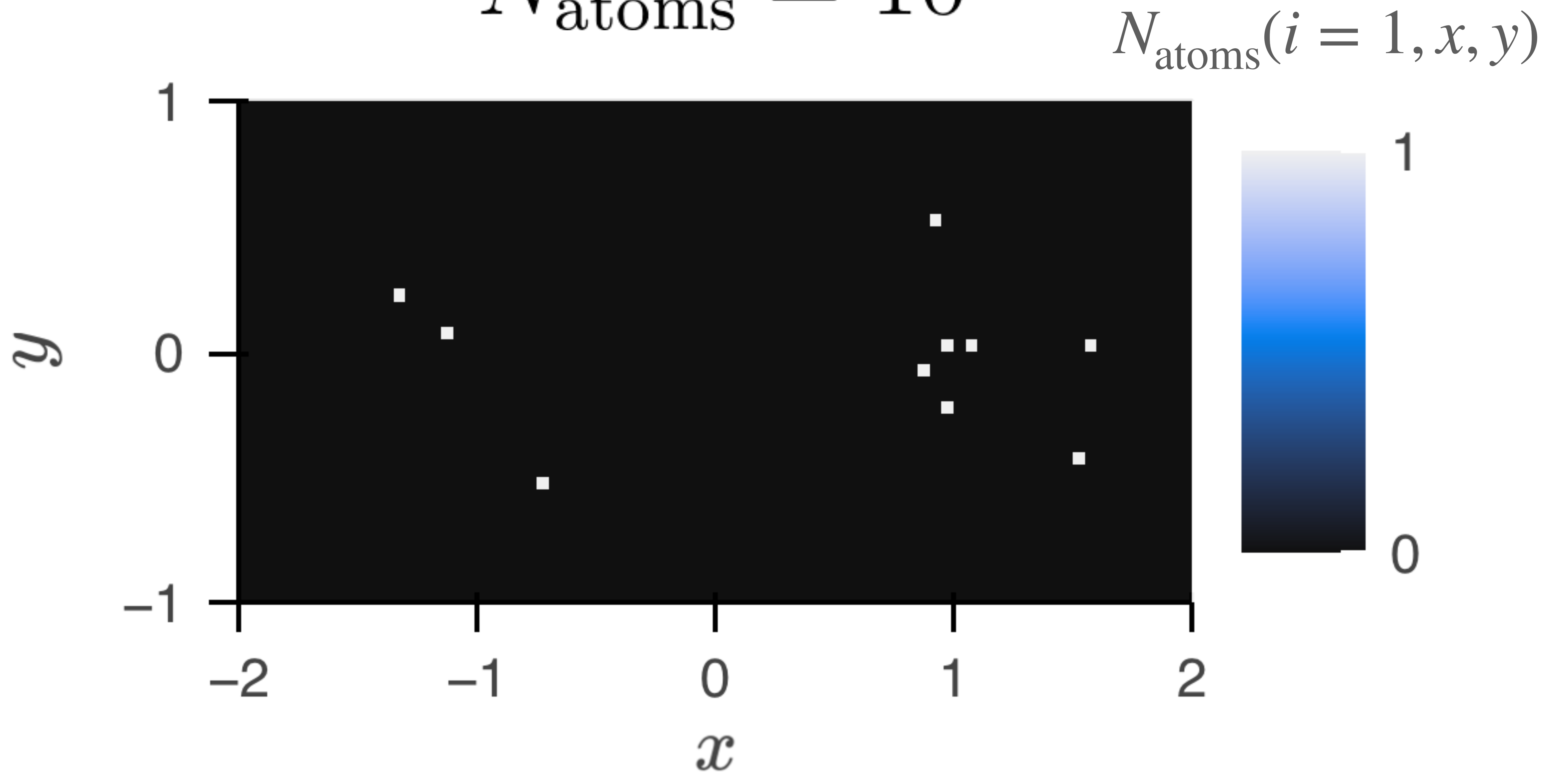
Spatial probability distribution

$P(x, y)$



Interferogram

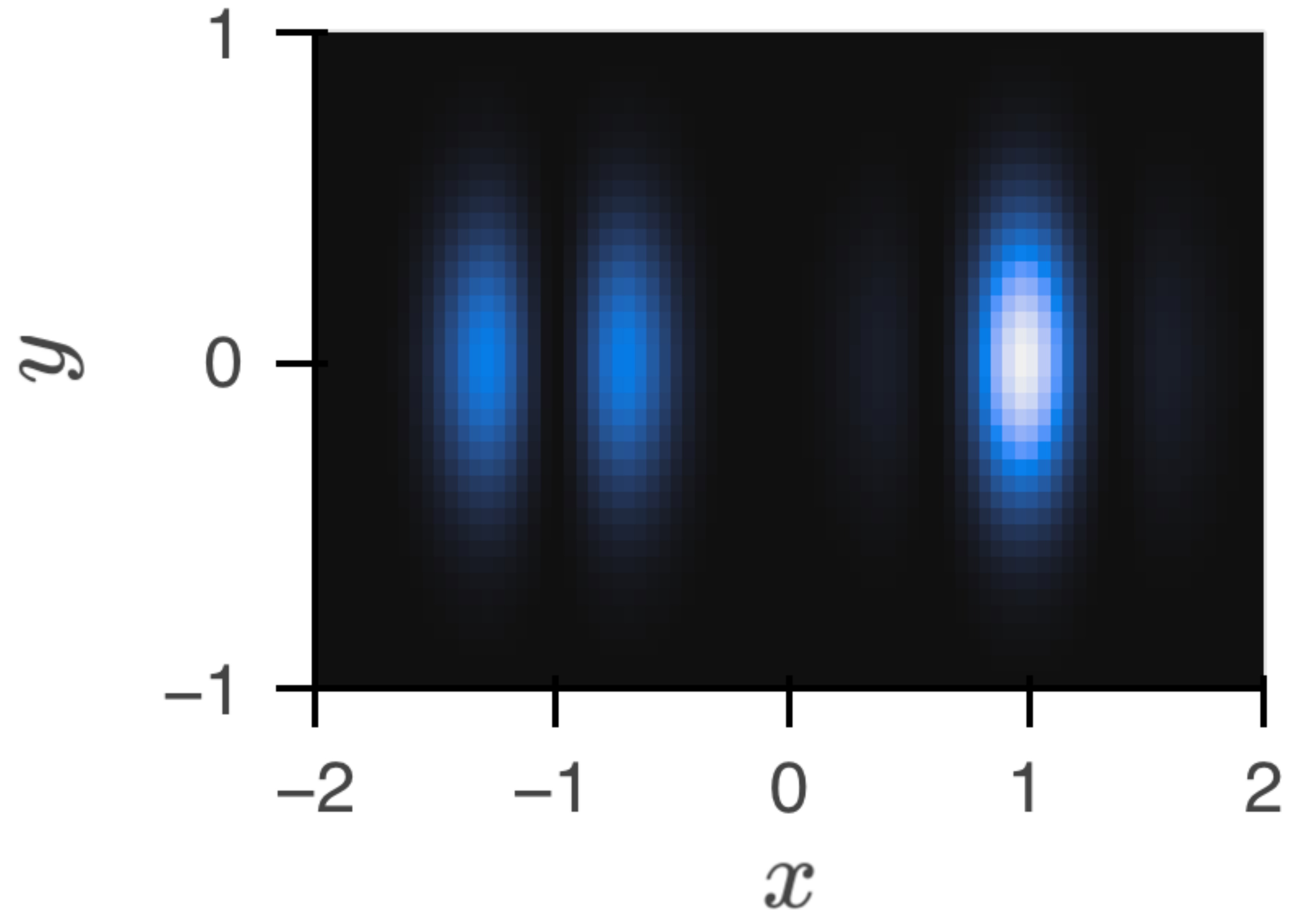
$N_{\text{atoms}} = 10$



Quantum Projection Noise (QPN)

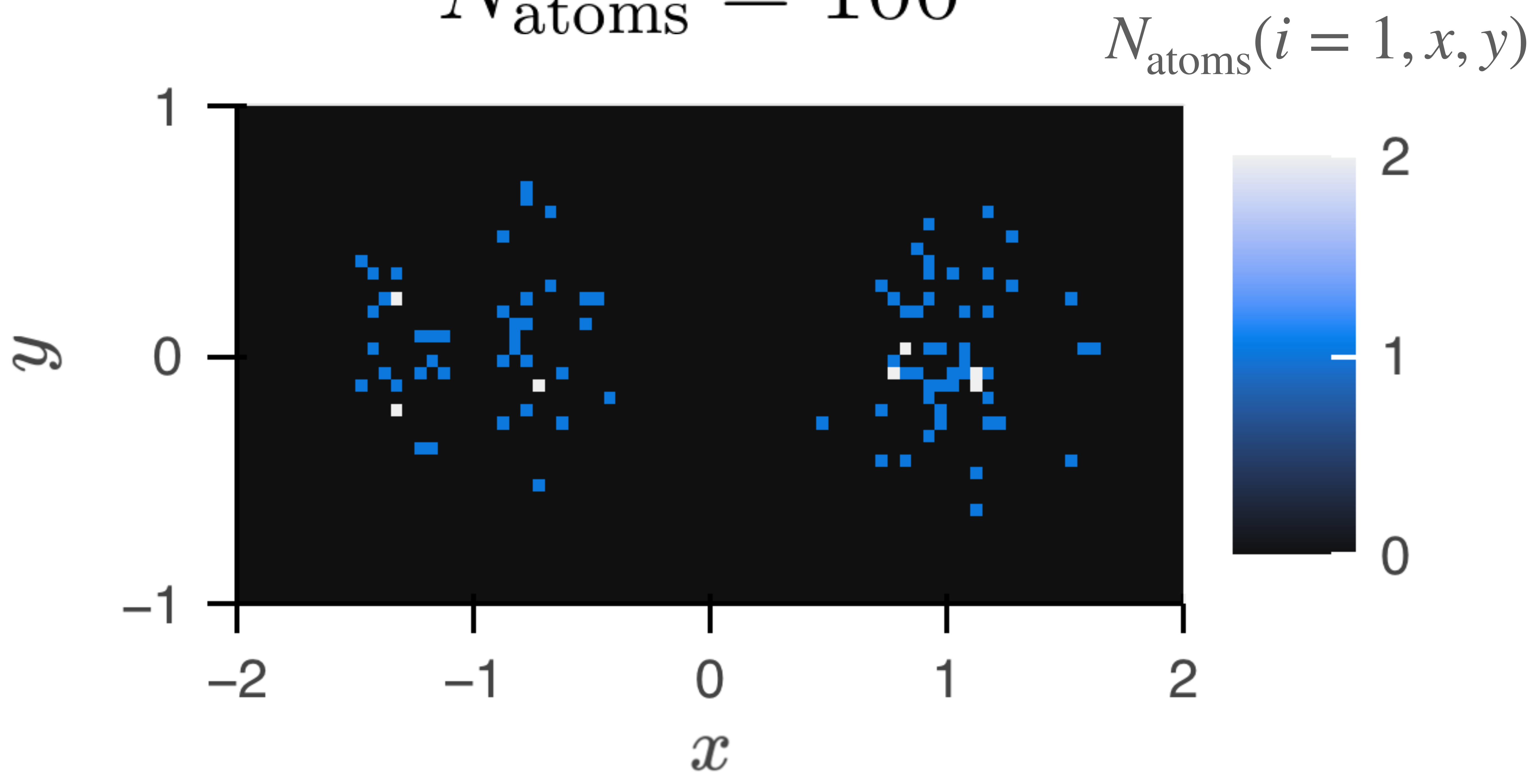
Spatial probability distribution

$P(x, y)$



Interferogram

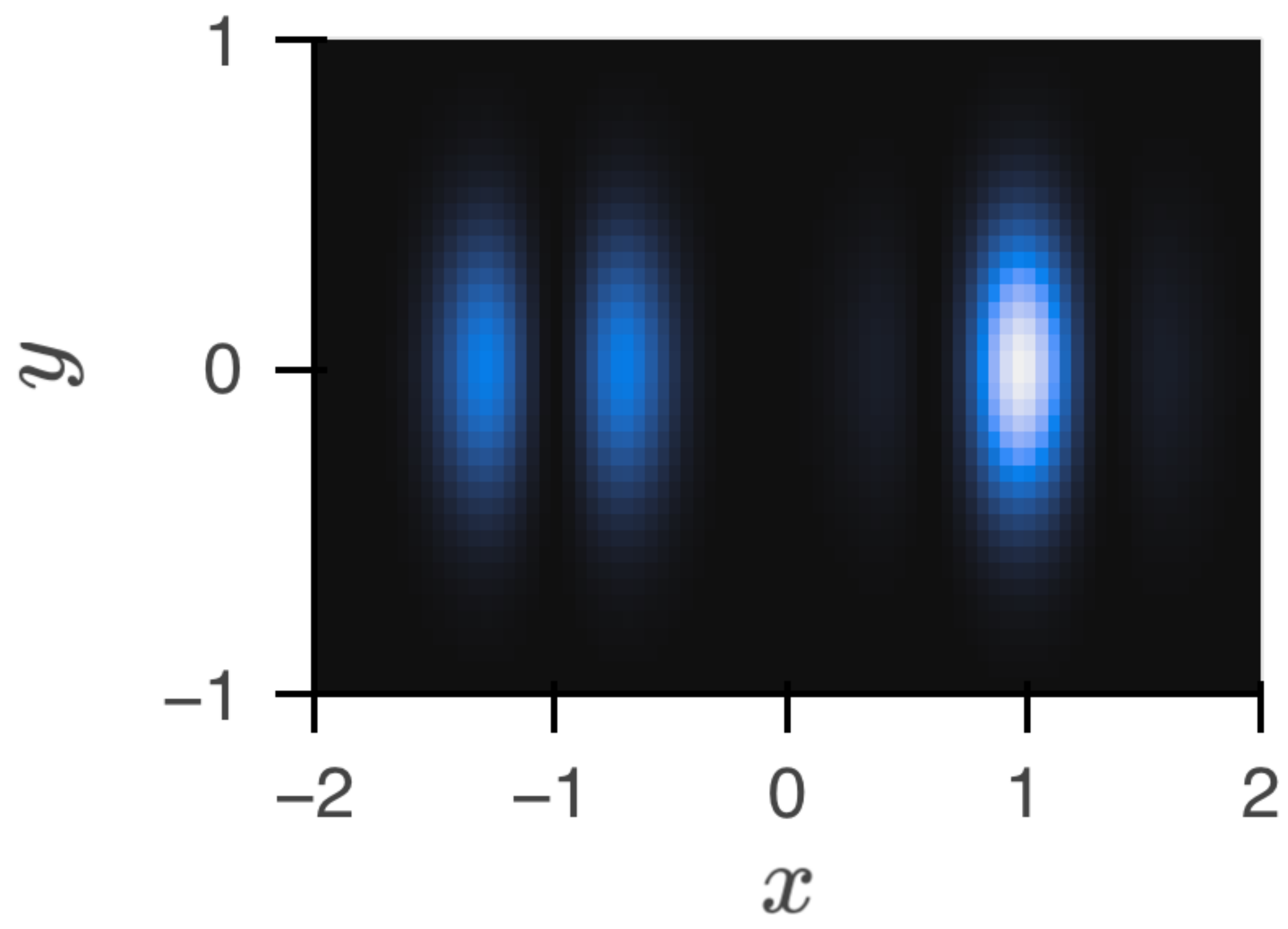
$N_{\text{atoms}} = 100$



Quantum Projection Noise (QPN)

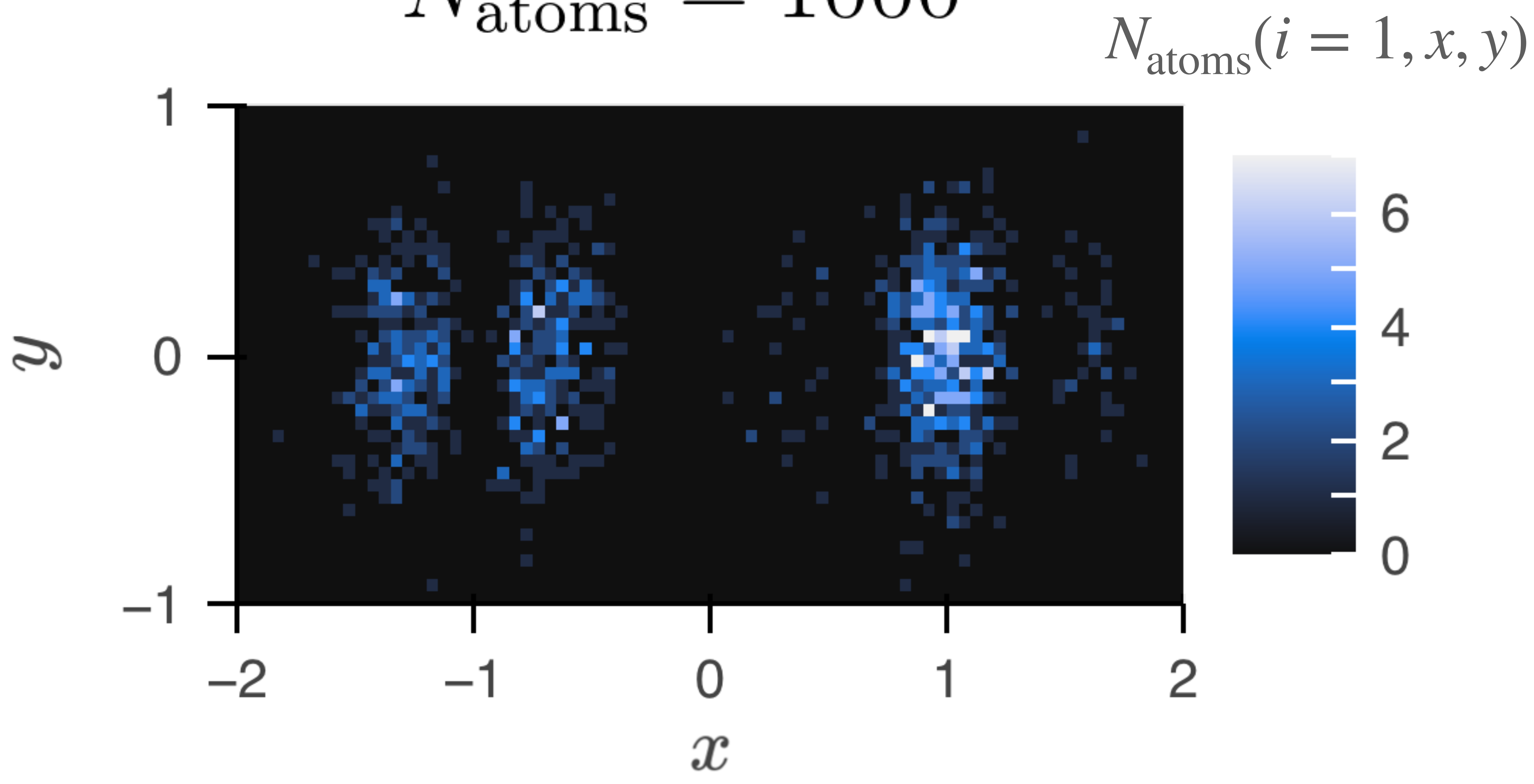
Spatial probability distribution

$$P(x, y)$$



Interferogram

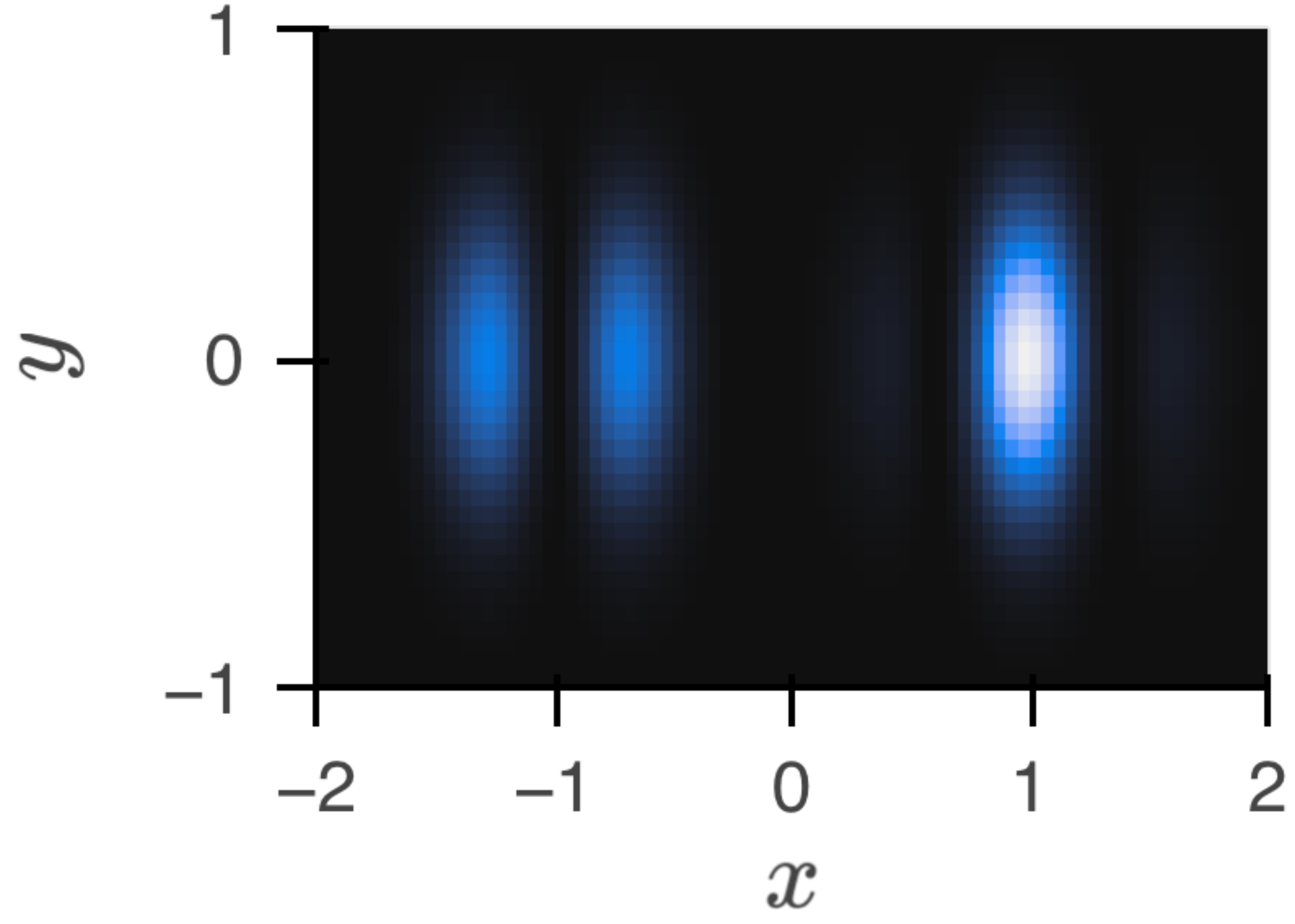
$$N_{\text{atoms}} = 1000$$



Quantum Projection Noise (QPN)

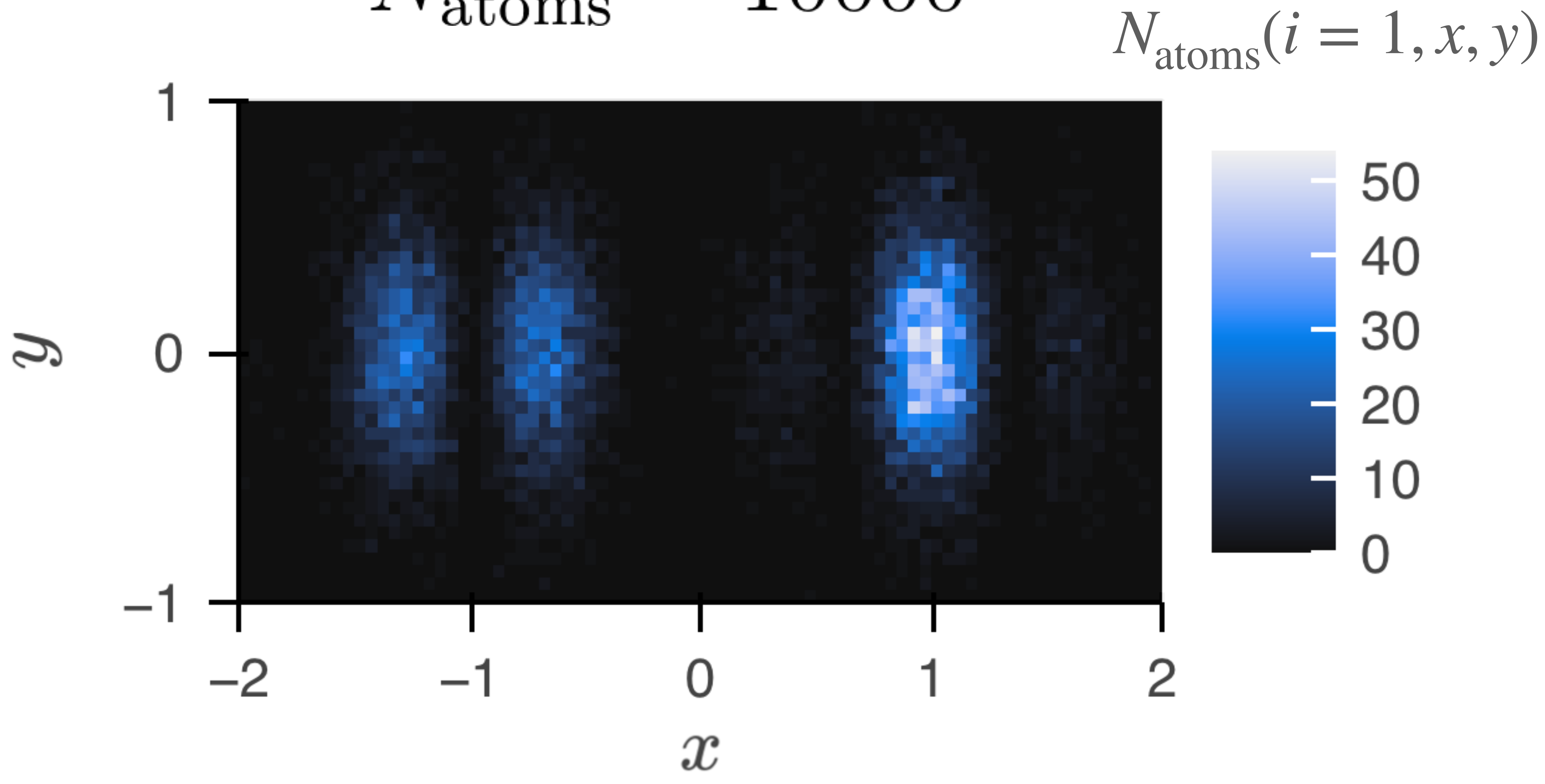
Spatial probability distribution

$$P(x,y)$$



Interferogram

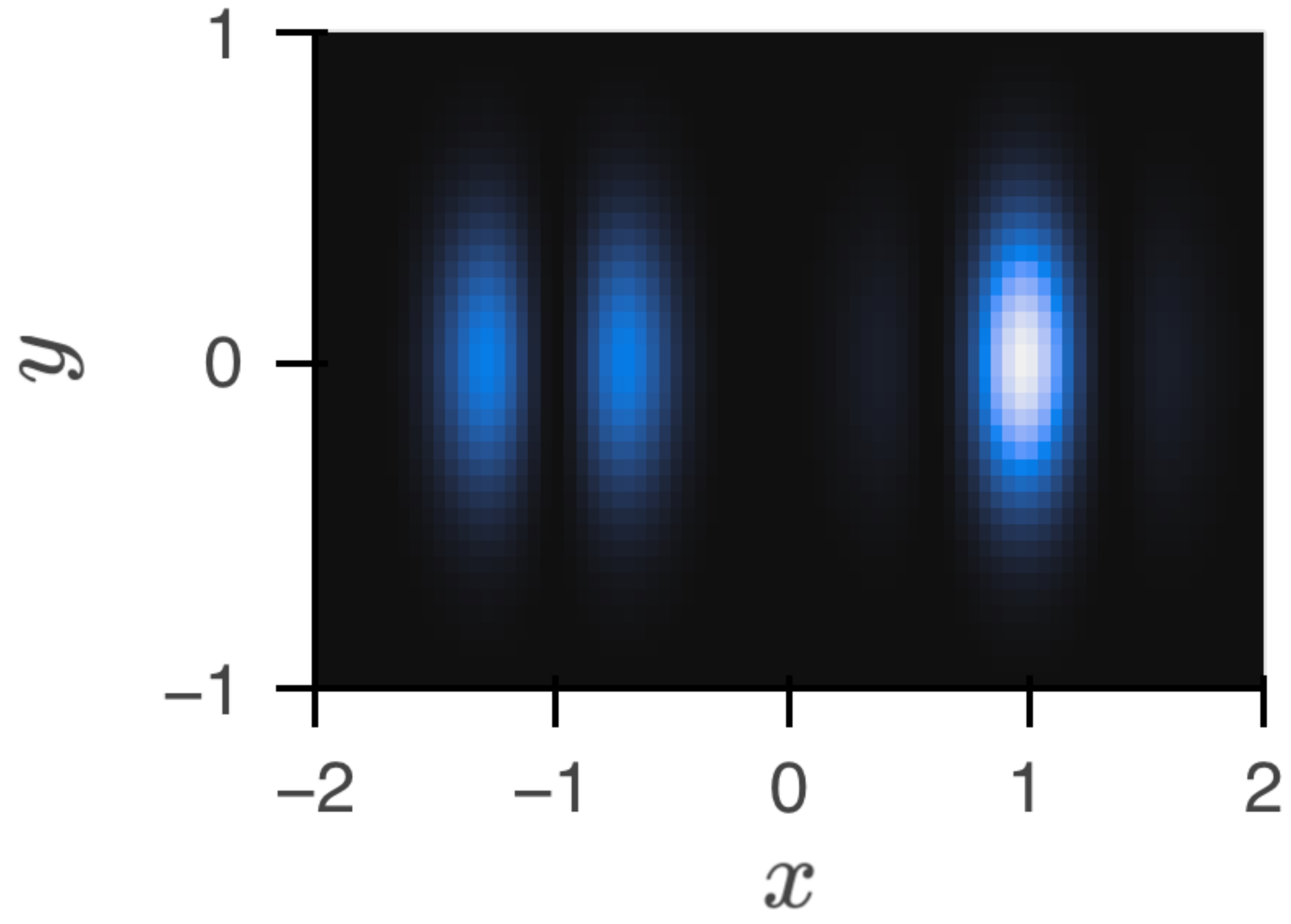
$$N_{\text{atoms}} = 10000$$



Quantum Projection Noise (QPN)

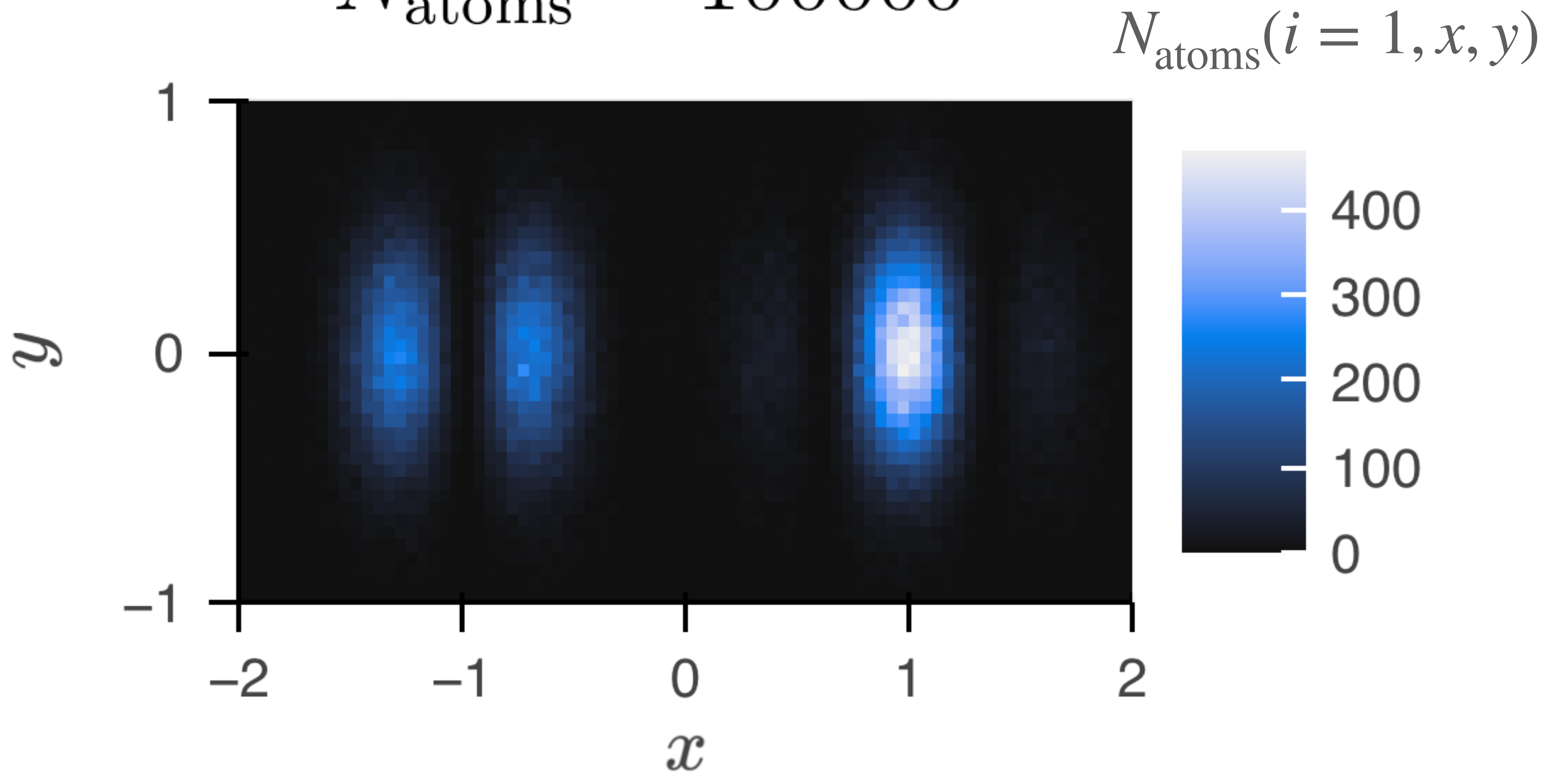
Spatial probability distribution

$$P(x, y)$$



Interferogram

$$N_{\text{atoms}} = 100000$$



Reconstruction Accuracy

- Assess statistical effects due to QPN
- Study three distinct spatial phase patterns

- Assess statistics of the reconstruction errors

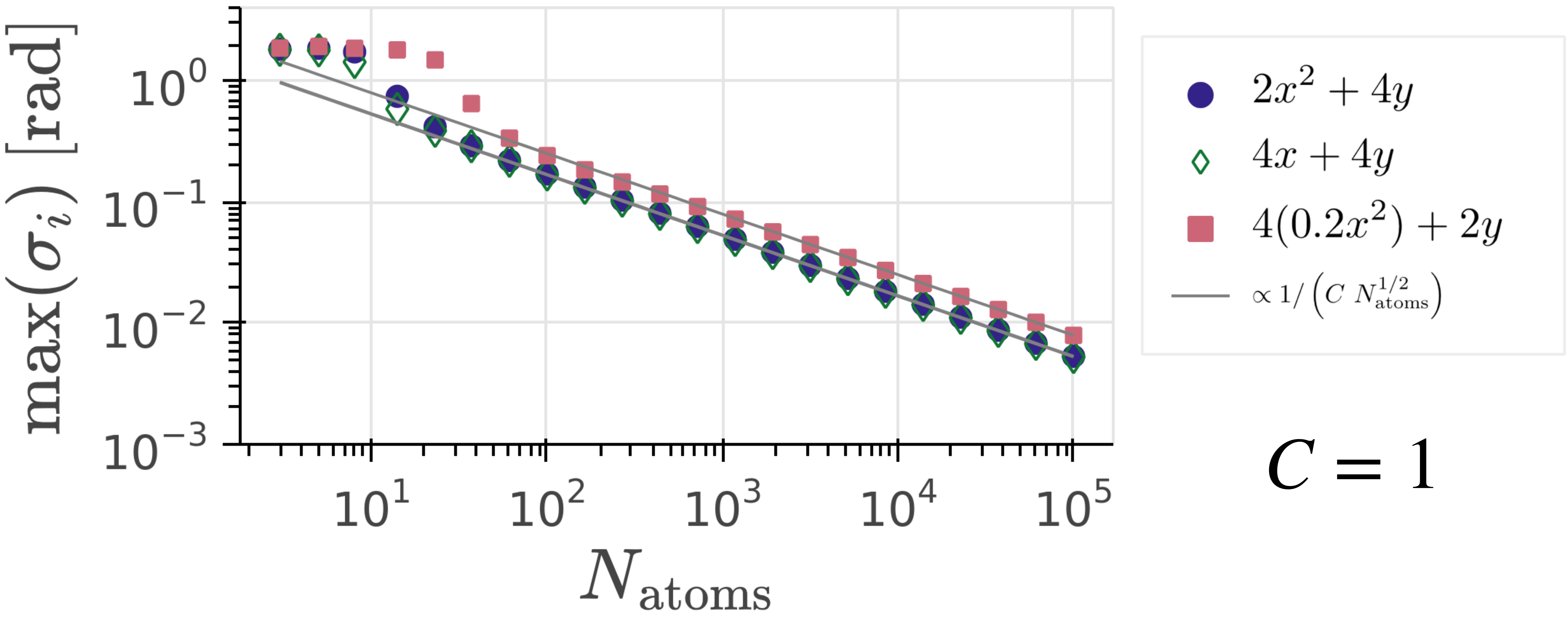
$$\theta_{\mathrm{diff}}(i) = \theta_{\mathrm{inp}}(i) - \theta_{\mathrm{rec}}(i) \qquad \gamma_{\mathrm{diff}}(x, y) = \gamma_{\mathrm{inp}}(x, y) - \gamma_{\mathrm{rec}}(x, y)$$

Reconstruction Accuracy: Per-Image Phase

- Average reconstruction error: $\text{avg}[\theta_{\text{diff}}(i)] \longrightarrow 0$
- Statistical uncertainty of reconstruction error (Error bar due to QPN) $\sigma_i \equiv \text{std}[\theta_{\text{diff}}(i)]$

$$\max(\sigma_i) \propto \frac{1}{C\ N_{\text{atoms}}^{1/2}}$$

Max phase uncertainty



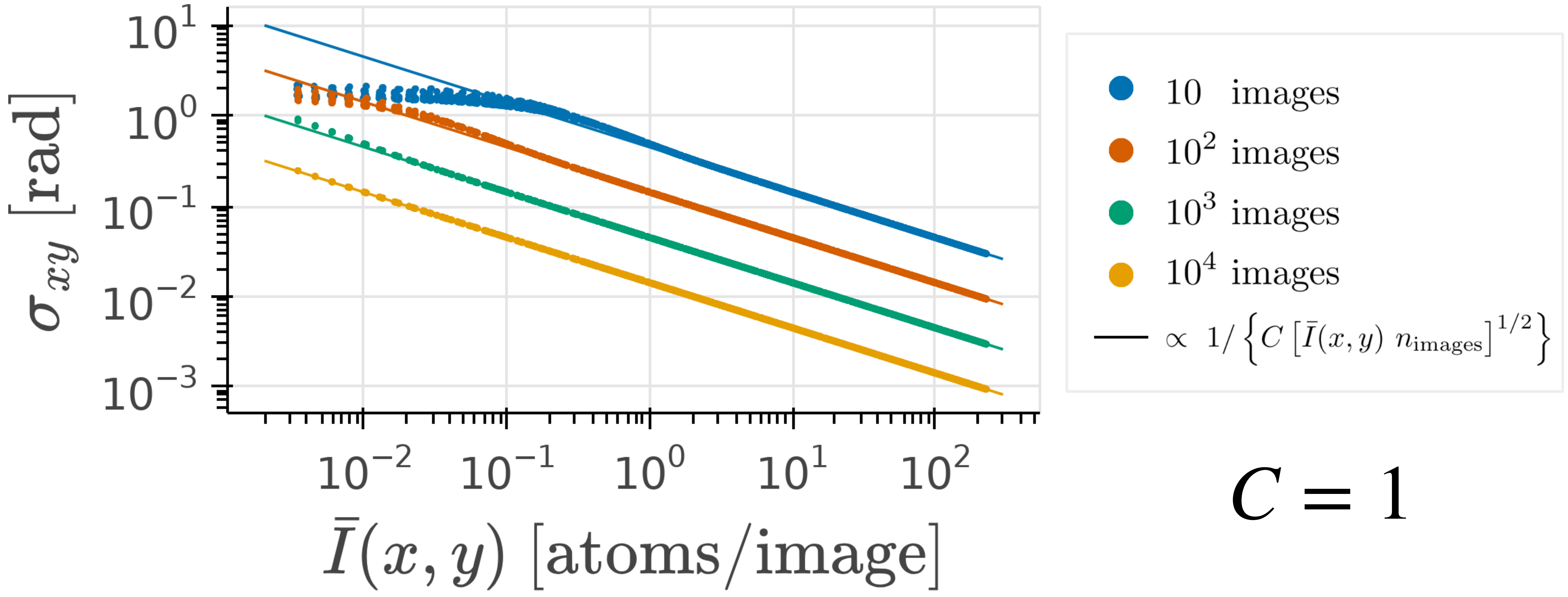
$C = 1$

Reconstruction Accuracy: Per-Pixel Phase

- Average reconstruction error: $\text{avg}[\gamma_{\text{diff}}(x,y)] \longrightarrow 0$
- Statistical uncertainty of reconstruction error (Error bar due to QPN) $\sigma_{xy} \equiv \text{std}[\gamma_{\text{diff}}(x,y)]$

$$\sigma_{xy} \propto \frac{1}{C\left[\bar{I}(x,y)\ n_{\text{images}}\right]^{1/2}}$$

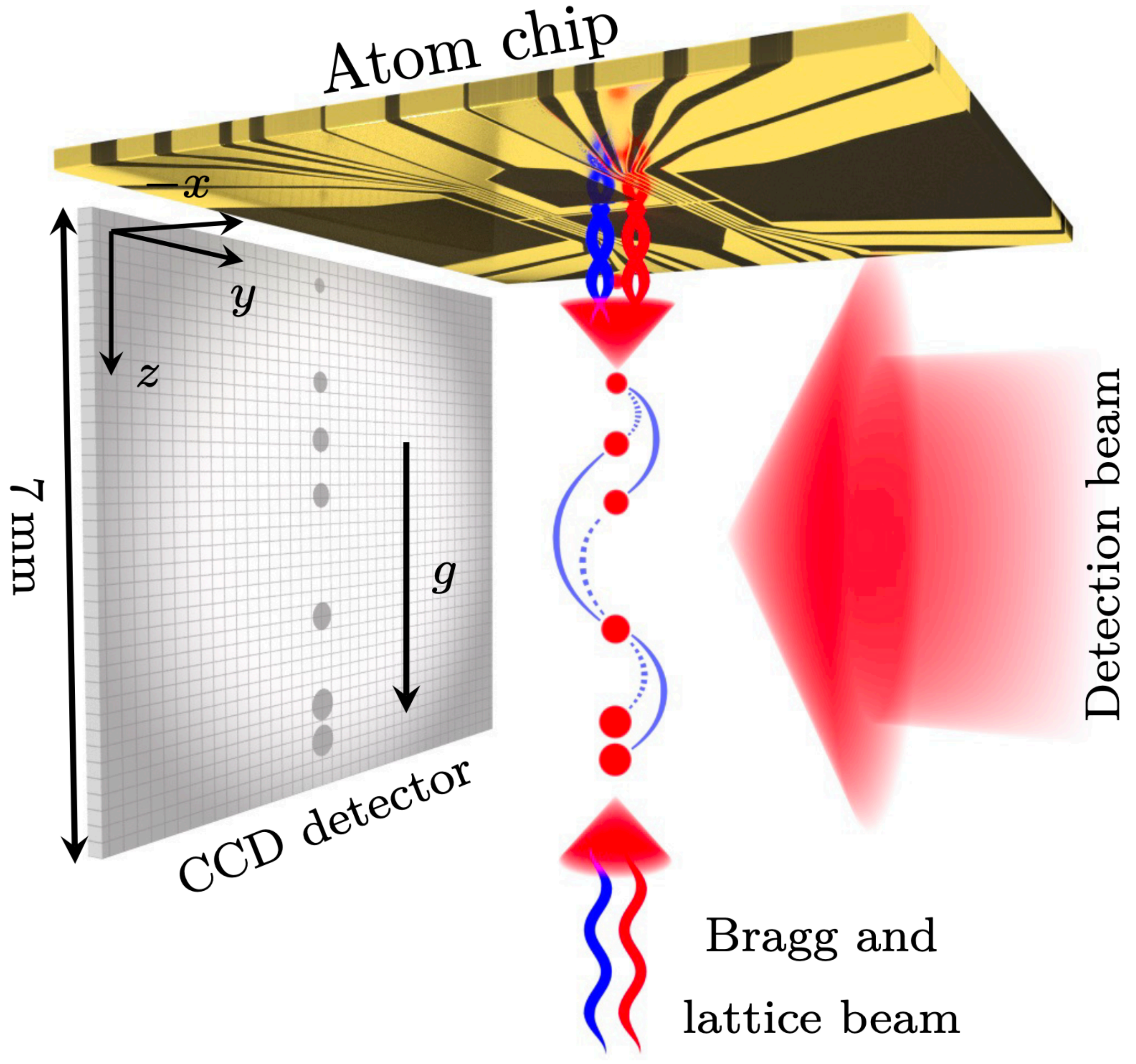
$\bar{I}$: Intensity averaged over i



$C = 1$

Application to Experiment

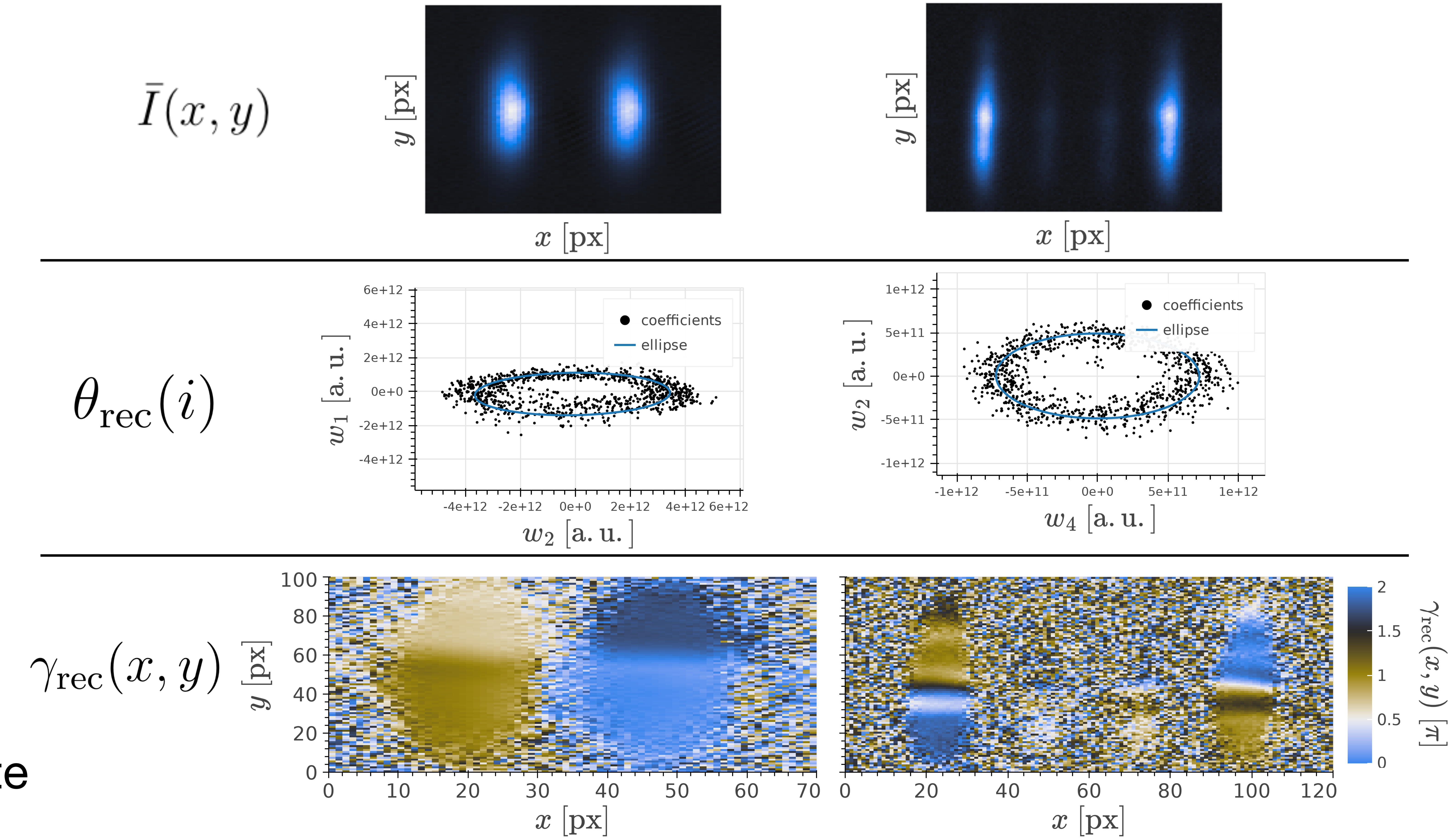
Chip gravimeter



Courtesy of S. Abend et al,
"Atom-chip fountain gravimeter,"
Phys. Rev. Lett. 117, 203003 (2016).

- Collimated Bose-Einstein condensate
- Higher-order Bragg interferometry

Momentum splits: $2\hbar k$ $6\hbar k$

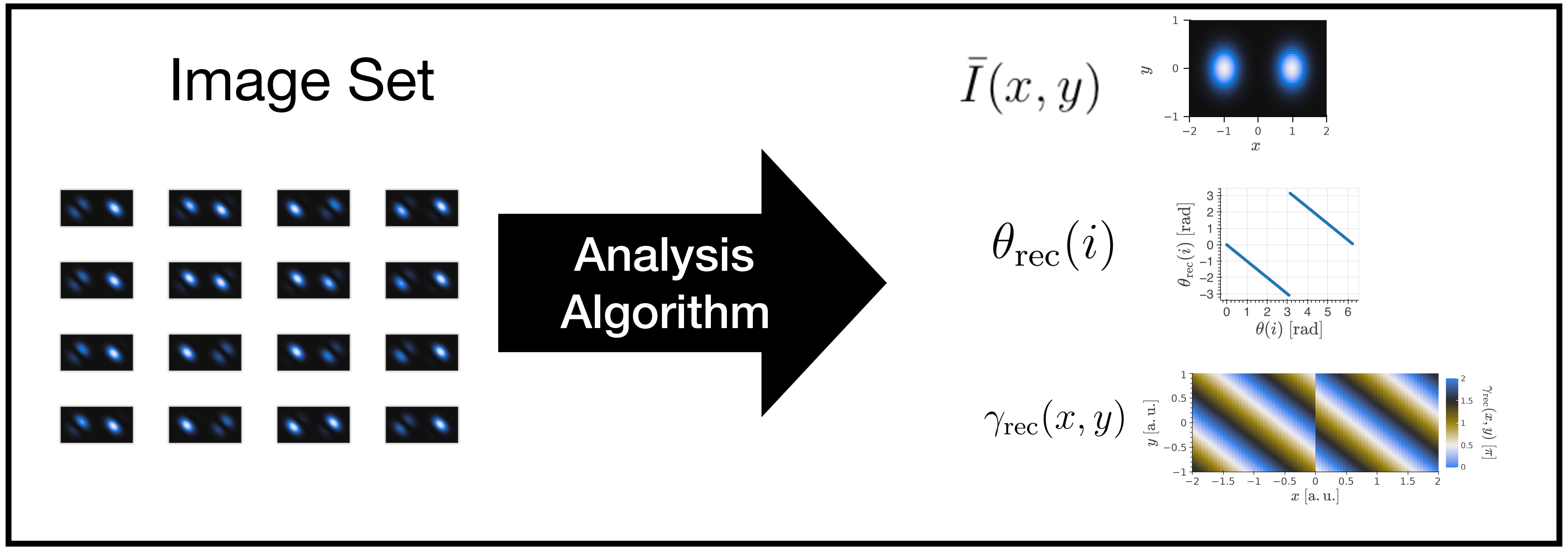


Spatial structures likely due to imperfections on chip coating

Summary

Spatially resolved phase reconstruction algorithm

Enhanced phase extraction algorithm



- Per image phase ("interferometer phase")
- Arbitrary static spatial intensity and phase profiles

- Robust to quantum projection noise
  - Up to mrad-accuracy in phase

- A tool to understand wavefront aberrations

Prospects

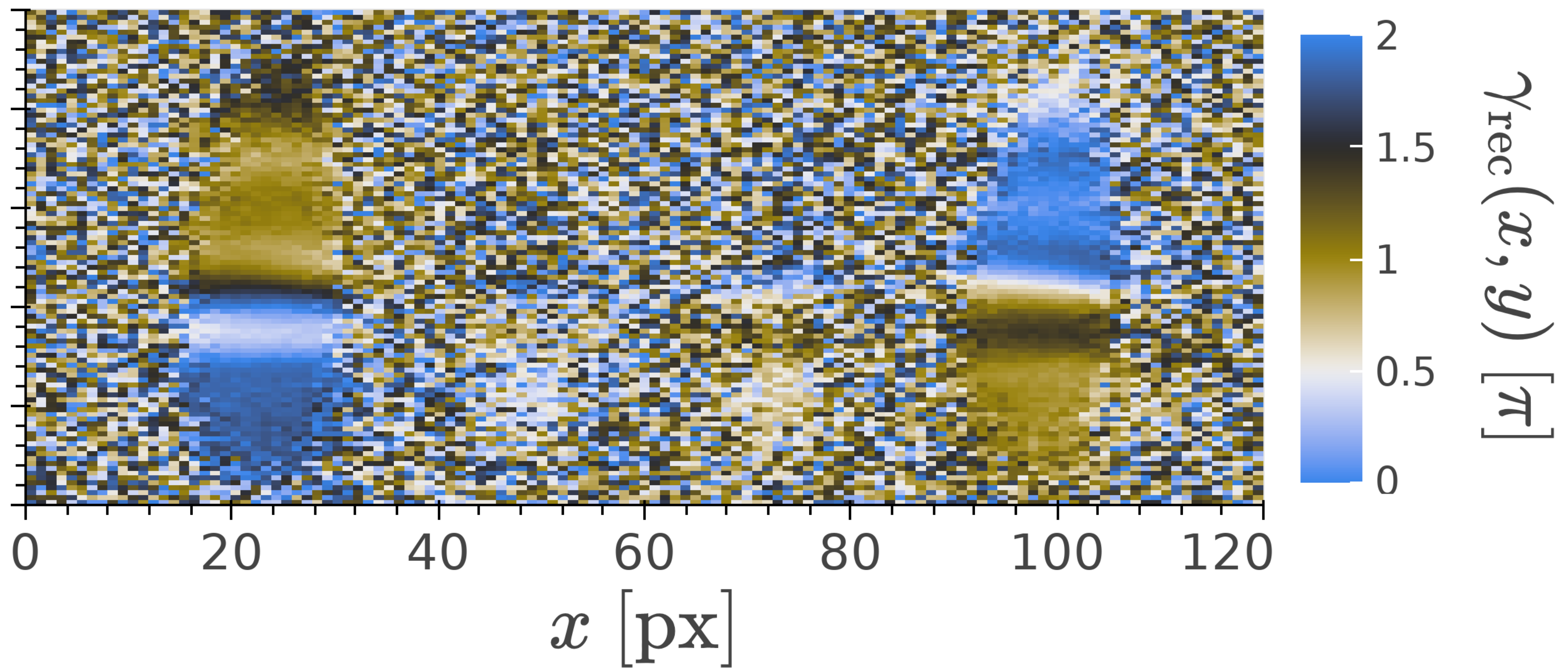
Spatially resolved phase reconstruction algorithm

Enhanced phase extraction algorithm

- Mitigating systematic shifts due to wavefront aberrations
- Measuring wavefront aberrations in-situ using the atoms

Data analysis tool suitable for autonomous operation

- Automatic tracking of phase profiles
- Resource efficient implementation



More ideas? Talk to us!

Thank You For Listening!

Stefan Seckmeyer

Theory of Quantum Sensing Group
by Naceur Gaaloul

Experimental work:

Group of Ernst M. Rasel